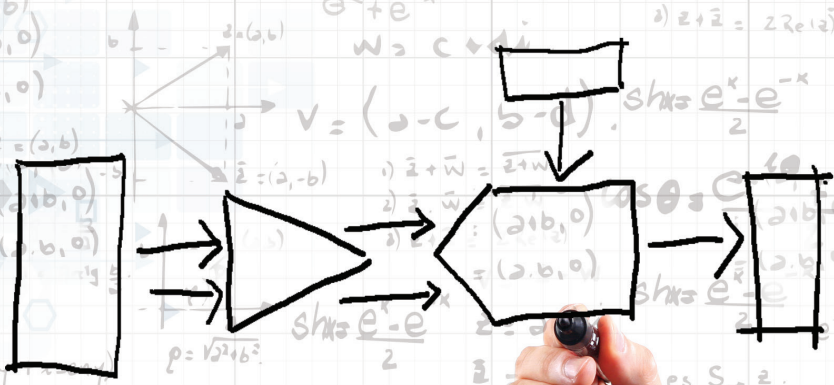


# Analog Engineer's

## Pocket Reference

Art Kay and Tim Green, Editors



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# Analog Engineer's Pocket Reference

Third Edition

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### *Message from the editors:*

This pocket reference is intended as a valuable quick guide for often used board- and system-level design formulae. This collection of formulae is based on a combined 50 years of analog board- and system-level expertise. Much of the material herein was referred to over the years via a folder stuffed full of printouts. Those worn pages have been organized and the information is now available via this guide in a bound and hard-to-lose format!

Here is a brief overview of the key areas included:

- Key constants and conversions
- Discrete components
- AC and DC analog equations
- Op amp basic configurations
- OP amp bandwidth and stability
- Overview of sensors
- PCB trace R, L, C
- Wire L, R, C
- Binary, hex and decimal formats
- A/D and D/A conversions

We hope you find this collection of formulae as useful as we have. Please send any comments and/or ideas you have for the next edition of the *Analog Engineer's Pocket Reference* to [artkay\\_timgreen@list.ti.com](mailto:artkay_timgreen@list.ti.com)

Additional resources:

- Search for complete board-and-system level circuits in the TI Designs – Precision reference design library ([www.ti.com/precisiondesigns](http://www.ti.com/precisiondesigns)).
- Read how-to blogs from TI precision analog experts at the Precision Hub ([www.ti.com/thehub](http://www.ti.com/thehub)).
- Find solutions, get help, share knowledge and solve problems with fellow engineers and TI experts in the TI E2E™ Community ([www.ti.com/e2e](http://www.ti.com/e2e)).

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- Standard decimal prefixes
- Metric conversions
- Temperature scale conversions
- Error conversions (ppm and percentage)



## Conversions

**Table 1: Physical constants**

Constant	Symbol	Value	Units
Speed of light in a vacuum	c	$3.00 \times 10^8$	m/s
Permittivity of vacuum	$\epsilon_0$	$8.85 \times 10^{-12}$	F/m
Permeability of free space	$\mu_0$	$1.26 \times 10^{-6}$	H/m
Plank's constant	h	$6.63 \times 10^{-34}$	Js
Boltzmann's constant	k	$1.38 \times 10^{-23}$	J/K
Faraday's constant	F	$9.65 \times 10^4$	C/mol
Avogadro's constant	$N_A$	$6.02 \times 10^{23}$	/mol
Unified atomic mass unit	$m_u$	$1.66 \times 10^{-27}$	kg
Electronic charge	q	$1.60 \times 10^{-19}$	C
Rest mass of electron	$m_e$	$9.11 \times 10^{-31}$	kg
Mass of proton	$m_p$	$1.67 \times 10^{-27}$	kg
Gravitational constant	G	$6.67 \times 10^{-11}$	$\text{Nm}^2/\text{kg}$
Standard gravity	$g_n$	9.81	$\text{m/s}^2$
Ice point	$T_{\text{ice}}$	273.15	K
Maximum density of water	$\rho$	$1.00 \times 10^3$	$\text{kg/m}^3$
Density of mercury	$\rho_{\text{Hg}}$	$1.36 \times 10^4$	$\text{kg/m}^3$
Gas constant	R	8.31	$\text{J}/(\text{K}\cdot\text{mol})$
Speed of sound in air (at 273 K)	$C_{\text{air}}$	$3.31 \times 10^2$	m/s



**Table 2: Standard decimal prefixes**

Multiplier	Prefix	Abbreviation
$10^{12}$	tera	T
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	k
$10^{-3}$	milli	m
$10^{-6}$	micro	$\mu$
$10^{-9}$	nano	n
$10^{-12}$	pico	p
$10^{-15}$	femto	f
$10^{-18}$	atto	a

**Table 3: English to metric conversions**

Unit	Symbol	Equivalent	Unit	Symbol
inches	in	25.4 mm/in	millimeter	mm
mil	mil	0.0254 mm/mil	millimeter	mm
feet	ft	0.3048 m/ft	meters	m
yards	yd	0.9144 m/yd	meters	m
miles	mi	1.6093 km/mi	kilometers	km
circular mil	cir mil	$5.067 \times 10^{-4} \text{ mm}^2/\text{cir mil}$	square millimeters	$\text{mm}^2$
square yards	$\text{yd}^2$	$0.8361 \text{ m}^2$	square meters	$\text{m}^2$
pints	pt	0.5682 L/pt	liters	L
ounces	oz	28.35 g/oz	grams	g
pounds	lb	0.4536 kg/lb	kilograms	kg
calories	cal	4.184 J/cal	joules	J
horsepower	hp	745.7 W/hp	watts	W

**Table 4: Metric to English conversions**

Unit	Symbol	Conversion	Unit	Symbol
millimeter	mm	0.0394 in/mm	inch	in
millimeter	mm	39.4 mil/mm	mil	mil
meters	m	3.2808 ft/m	feet	ft
meters	m	1.0936 yd/m	yard	yd
kilometers	km	0.6214 mi/km	miles	mi
square millimeters	$\text{mm}^2$	$1974 \text{ cir mil}/\text{mm}^2$	circular mil	cir mil
square meters	$\text{m}^2$	$1.1960 \text{ yd}^2/\text{m}^2$	square yards	$\text{yd}^2$
liters	L	1.7600 pt/L	pints	pt
grams	g	0.0353 oz/g	ounces	oz
kilograms	kg	2.2046 lb/kg	pounds	lb
joules	J	0.239 cal/J	calories	cal
watts	W	$1.341 \times 10^{-3} \text{ hp}/\text{W}$	horsepower	hp

**Example**

Convert 10 mm to mil.

**Answer**

$$10 \text{ mm} \times 39.4 \frac{\text{mil}}{\text{mm}} = 384 \text{ mil}$$

# Conversions

---

**Table 5: Temperature conversions**

$^{\circ}\text{C} = \frac{5}{9} (^{\circ}\text{F} - 32)$	Fahrenheit to Celsius
$^{\circ}\text{F} = \frac{9}{5} (^{\circ}\text{C}) + 32$	Celsius to Fahrenheit
$\text{K} = ^{\circ}\text{C} + 273.15$	Celsius to Kelvin
$^{\circ}\text{C} = \text{K} - 273.15$	Kelvin to Celsius

**Table 6: Error conversions**

$\text{Error}(\%) = \frac{\text{Measured} - \text{Ideal}}{\text{Ideal}} \times 100$	Error in measured value
$\text{Error}(\% \text{ FSR}) = \frac{\text{Measured} - \text{Ideal}}{\text{Full-scale range}} \times 100$	Error in percent of full-scale range
$\% = \frac{\text{ppm}}{10^6} \times 100$	Part per million to percent
$\text{m}\% = \frac{\text{ppm}}{10^6} \times 100 \times 1000$	Part per million to milli-percent
$\text{ppm} = \% \times 10^4$	Percent to part per million
$\text{ppm} = \text{m}\% \times 10$	Milli-percent to part per million

## Example

Compute the error for a measured value of 0.12V when the ideal value is 0.1V and the range is 5V.

## Answer

$\text{Error}(\%) = \frac{0.12\text{V} - 0.1\text{V}}{0.1\text{V}} \times 100 = 20\%$	Error in measured value
$\text{Error}(\% \text{ FSR}) = \frac{0.12 - 0.1\text{V}}{5\text{V}} \times 100 = 0.4\%$	Percent FSR

## Example

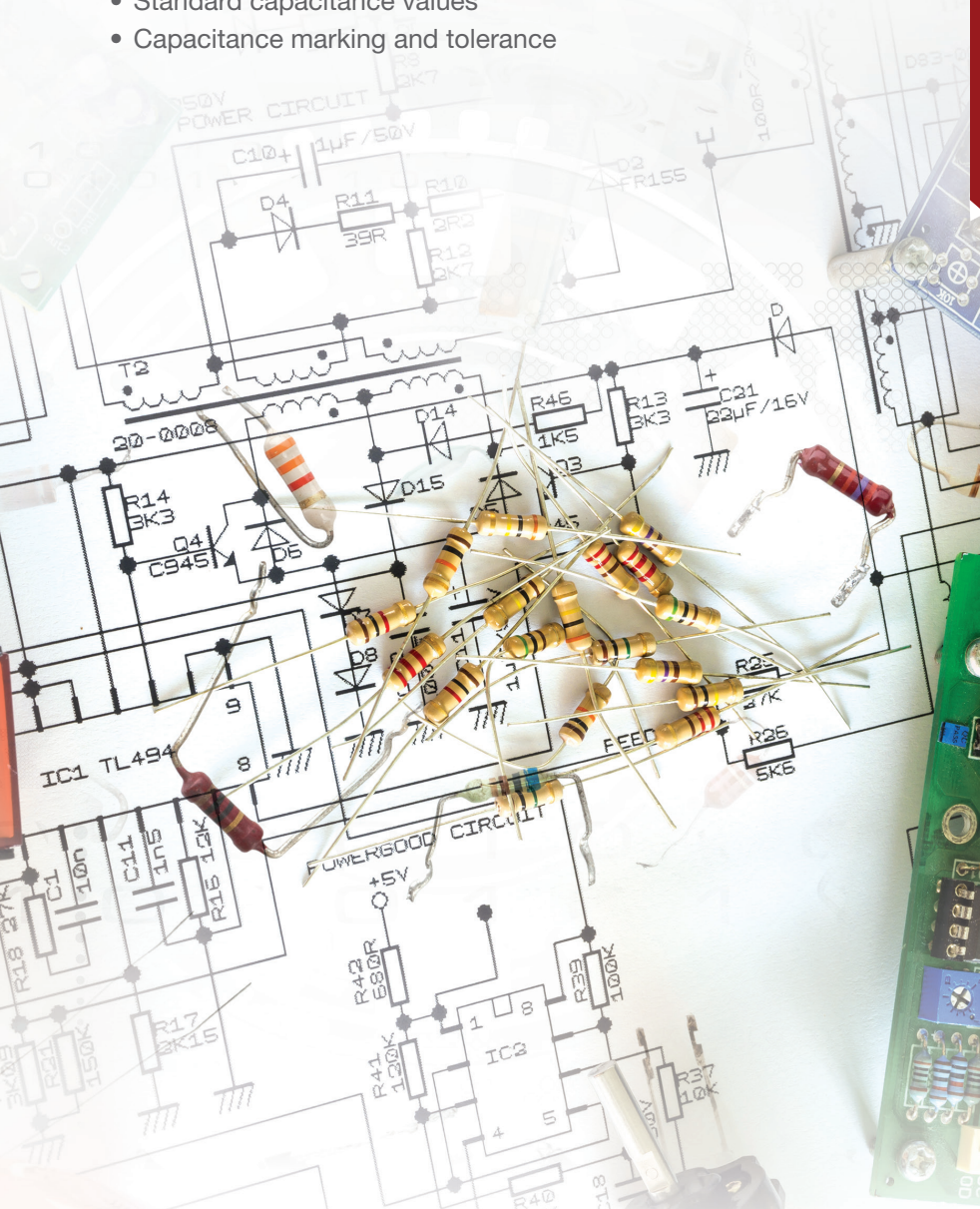
Convert 10 ppm to percent and milli-percent.

## Answer

$\frac{10 \text{ ppm}}{10^6} \times 100 = 0.001\%$	Part per million to percent
$\frac{10 \text{ ppm}}{10^6} \times 100 \times 1000 = 1 \text{ m}\%$	Part per million to milli-percent

## Discrete components

- Resistor color code
- Standard resistor values
- Capacitance specifications
- Capacitance type overview
- Standard capacitance values
- Capacitance marking and tolerance



## Discrete components

Table 7: Resistor color code

Color	Digit	Additional Zeros	Tolerance
None	-na-	-na-	20%
Silver	-na-	-2	10%
Gold	-na-	-1	5%
Black	0	0	
Brown	1	1	
Red	2	2	2%
Orange	3	3	
Yellow	4	4	
Green	5	5	
Blue	6	6	
Violet	7	7	
Grey	8		
White	9		

### Example

Yellow, violet, orange and silver indicate 4, 7, and 3 zeros. or a 47 k $\Omega$ , 10% resistor.

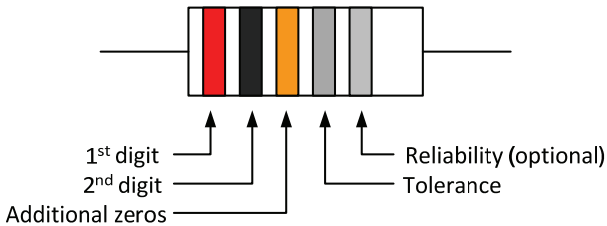


Figure 1: Resistor color code

**Table 8: Standard resistor values**

		Standard resistance values for the 10 to 100 decade																							
		0.1%			2%			0.1%			2%			0.1%			2%								
		0.25%	1%	5%	0.25%	1%	5%	0.25%	1%	5%	0.25%	1%	5%	0.25%	1%	5%	0.25%	1%	5%						
10.0	10.0	14.7	14.7	21.5	21.5	31.6	31.6	46.4	46.4	68.1	68.1	100	100	10	10	10	10	10	10	10	10	10	68	68	
10.1	10.1	14.9	14.9	21.8	21.8	32.0	32.0	47.0	47.0	69.0	69.0	10.2	10.2	15	15	22	22	32.4	32.4	47.5	47.5	69.8	69.8	69.8	69.8
10.4	10.4	15.2	15.2	22.3	22.3	32.8	32.8	48.1	48.1	70.6	70.6	10.5	10.5	15.4	15.4	22.6	22.6	33.2	33.2	48.7	48.7	71.5	71.5	71.5	71.5
10.6	10.6	15.6	15.6	22.9	22.9	33.6	33.6	49.3	49.3	72.3	72.3	10.7	10.7	15.8	15.8	23.2	23.2	34.0	34.0	49.9	49.9	73.2	73.2	73.2	73.2
10.9	10.9	16.0	16.0	23.4	23.4	34.4	34.4	50.5	50.5	74.1	74.1	11.0	11.0	16.2	16.2	23.7	23.7	34.8	34.8	51.1	51.1	75.0	75.0	75	75
11.1	11.1	16.4	16.4	24.0	24.0	35.2	35.2	51.7	51.7	75.9	75.9	11.3	11.3	16.5	16.5	24.3	24.3	35.7	35.7	52.3	52.3	76.8	76.8	76.8	76.8
11.4	11.4	16.7	16.7	24.6	24.6	36.1	36.1	53.0	53.0	77.7	77.7	11.5	11.5	16.9	16.9	24.9	24.9	36.5	36.5	53.6	53.6	78.7	78.7	78.7	78.7
11.7	11.7	17.2	17.2	25.2	25.2	37.0	37.0	54.2	54.2	79.6	79.6	11.8	11.8	17.4	17.4	25.5	25.5	37.4	37.4	54.9	54.9	80.6	80.6	80.6	80.6
12.0	12.0	17.6	17.6	25.8	25.8	37.9	37.9	55.6	55.6	81.6	81.6	12.1	12.1	17.8	17.8	26.1	26.1	38.3	38.3	56.2	56.2	82.5	82.5	82	82
12.3	12.3	18.0	18.0	26.4	26.4	38.8	38.8	56.9	56.9	83.5	83.5	12.4	12.4	18.2	18.2	26.7	26.7	39.2	39.2	57.6	57.6	84.5	84.5	84.5	84.5
12.6	12.6	18.4	18.4	27.1	27.1	39.7	39.7	58.3	58.3	85.6	85.6	12.7	12.7	18.7	18.7	27.4	27.4	40.2	40.2	59.0	59.0	86.6	86.6	86.6	86.6
12.9	12.9	18.9	18.9	27.7	27.7	40.7	40.7	59.7	59.7	87.6	87.6	13.0	13.0	19.1	19.1	28.0	28.0	41.2	41.2	60.4	60.4	88.7	88.7	88.7	88.7
13.0	13.0	19.1	19.1	28.0	28.0	41.7	41.7	61.2	61.2	89.8	89.8	13.2	13.2	19.3	19.3	28.4	28.4	42.2	42.2	61.9	61.9	90.9	90.9	90.9	90.9
13.3	13.3	19.6	19.6	28.7	28.7	42.7	42.7	62.6	62.6	92.0	92.0	13.5	13.5	19.8	19.8	29.1	29.1	43.2	43.2	63.4	63.4	93.1	93.1	93.1	93.1
13.7	13.7	20.0	20.0	29.4	29.4	43.7	43.7	64.2	64.2	94.2	94.2	13.8	13.8	20.3	20.3	29.8	29.8	44.2	44.2	64.9	64.9	95.3	95.3	95.3	95.3
14.0	14.0	20.5	20.5	30.1	30.1	44.8	44.8	65.7	65.7	96.5	96.5	14.2	14.2	20.8	20.8	30.5	30.5	45.3	45.3	66.5	66.5	97.6	97.6	97.6	97.6
14.3	14.3	21.0	21.0	30.9	30.9	45.9	45.9	66.5	66.5	97.6	97.6	14.5	14.5	21.3	21.3	31.2	31.2	46.4	46.4	68.1	68.1	98.8	98.8	98.8	98.8

## Discrete components

### Practical capacitor model and specifications

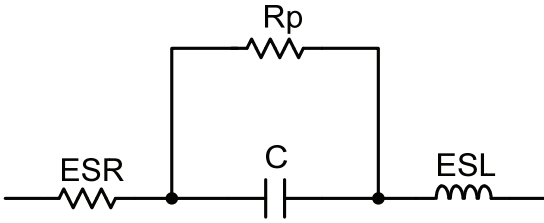


Figure 2: Model of a practical capacitor.

Table 9: Capacitor specifications

Parameter	Description
C	<i>The nominal value of the capacitance.</i> Table 11 lists standard capacitance values.
ESR	<i>Equivalent series resistance.</i> Ideally this is zero. Ceramic capacitors have the best ESR (typically in milliohms). Tantalum Electrolytic have ESR in the hundreds of milliohms and Aluminum Electrolytic have ESR in the ohms.
ESL	<i>Equivalent series inductance.</i> Ideally this is zero. ESL ranges from 100 pH to 10 nH.
Rp	<i>Rp is a parallel leakage resistance (or insulation resistance).</i> Ideally this is infinite. This can range from tens of megaohms for some electrolytic capacitors to tens of gigohms for ceramic.
Voltage rating	<i>The maximum voltage that can be applied to the capacitor.</i> Exceeding this rating damages the capacitor.
Voltage coefficient	<i>The change in capacitance with applied voltage in ppm/V.</i> A high-voltage coefficient can introduce distortion. C0G capacitors have the lowest coefficient. The voltage coefficient is most important in applications that use capacitors in signal processing such as filtering.
Temperature coefficient	<i>The change in capacitance with across temperature in ppm/°C.</i> Ideally, the temperature coefficient is zero. The maximum specified drift generally ranges from 10 to 100ppm/°C depending on the resistor type.

Practical capacitors vs. frequency

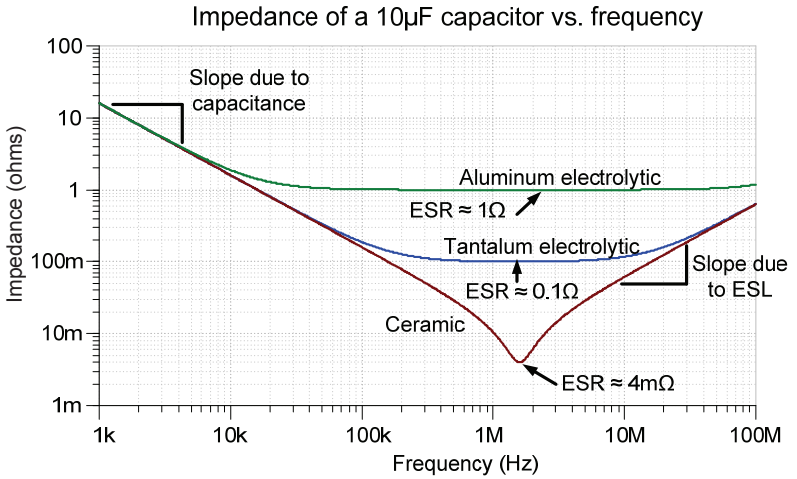


Figure 3: Effect of ESR and ESL on capacitor frequency response

## Discrete components

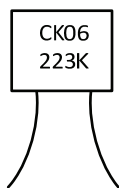
**Table 10: Capacitor type overview**

Capacitor type	Description
C0G/NP0 (Type 1 ceramic)	<p><i>Use in signal path, filtering, low distortion, audio, and precision</i></p> <p>Limited capacitance range: 0.1 pF to 0.47 <math>\mu</math>F            Lowest temperature coefficient: <math>\pm 30</math> ppm/<math>^{\circ}</math>C            Low-voltage coefficient            Minimal piezoelectric effect            Good tolerance: <math>\pm 1\%</math> to <math>\pm 10\%</math>            Temperature range: <math>-55^{\circ}</math>C to <math>125^{\circ}</math>C (<math>150^{\circ}</math>C and higher)            Voltage range may be limited for larger capacitance values</p>
X7R (Type 2 ceramic)	<p><i>Use for decoupling and other applications where accuracy and low distortion are not required</i></p> <p>X7R is an example of a type 2 ceramic capacitor            See <b>EIA capacitor tolerance table</b> for details on other types            Capacitance range: 10 pF to 47 <math>\mu</math>F            Temperature coefficient: <math>\pm 833</math> ppm/<math>^{\circ}</math>C (<math>\pm 15\%</math> across temp range)            Substantial voltage coefficient            Tolerance: <math>\pm 5\%</math> to <math>-20\%/+80\%</math>            Temperature range: <math>-55^{\circ}</math>C to <math>125^{\circ}</math>C            Voltage range may be limited for larger capacitance values</p>
Y5V (Type 2 ceramic)	<p><i>Use for decoupling and other applications where accuracy and low distortion are not required</i></p> <p>Y5V is an example of a type 2 ceramic capacitor            See <b>EIA capacitor tolerance table</b> for details on other types            Temperature coefficient: <math>-20\%/+80\%</math> across temp range            Temperature range: <math>-30^{\circ}</math>C to <math>85^{\circ}</math>C            Other characteristics are similar to X7R and other type 2 ceramic</p>
Aluminum oxide electrolytic	<p><i>Use for bulk decoupling and other applications where large capacitance is required</i></p> <p>Note that electrolytic capacitors are polarized and will be damaged, if a reverse polarity connection is made            Capacitance range: 1 <math>\mu</math>F to 68,000 <math>\mu</math>F            Temperature coefficient: <math>\pm 30</math> ppm/<math>^{\circ}</math>C            Substantial voltage coefficient            Tolerance: <math>\pm 20\%</math>            Temperature range: <math>-55^{\circ}</math>C to <math>125^{\circ}</math>C (<math>150^{\circ}</math>C and higher)            Higher ESR than other types</p>
Tantalum electrolytic	<p>Capacitance range: 1 <math>\mu</math>F to 150 <math>\mu</math>F            Similar to aluminum oxide but smaller size</p>
Polypropylene film	<p>Capacitance range: 100 pF to 10 <math>\mu</math>F            Very low voltage coefficient (low distortion)            Higher cost than other types            Larger size per capacitance than other types            Temperature coefficient: 2% across temp range            Temperature range: <math>-55^{\circ}</math>C to <math>100^{\circ}</math>C</p>



**Table 11: Standard capacitance table**

Standard capacitance table											
1	1.1	1.2	1.3	1.5	1.6	1.8	2	2.2	2.4	2.7	3
3.3	3.6	3.9	4.3	4.7	5.1	5.6	6.2	6.8	7.5	8.2	9.1



**Example**

Translate the capacitor marking

$$\begin{array}{l}
 \overbrace{223}^{\text{K}} \rightarrow \text{"K"} = \pm 10\% \\
 \downarrow \quad \downarrow \\
 22\ 000\ \text{pF} \\
 = 22\text{nF} = 0.022\mu\text{F}
 \end{array}$$

**Figure 4: Capacitor marking code**

**Table 12: Ceramic capacitor tolerance markings**

Code	Tolerance		Code	Tolerance
B	$\pm 0.1\ \text{pF}$		J	$\pm 5\%$
C	$\pm 0.25\ \text{pF}$		K	$\pm 10\%$
D	$\pm 0.5\ \text{pF}$		M	$\pm 20\%$
F	$\pm 1\%$		Z	+ 80%, -20%
G	$\pm 2\%$			

**Table 13: EIA capacitor tolerance markings (Type 2 capacitors)**

First letter symbol	Low temp. limit	Second number symbol	High temp. limit	Second letter symbol	Max. capacitance change over temperature rating
Z	+10°C	2	+45°C	A	$\pm 1.0\%$
Y	-30°C	4	+65°C	B	$\pm 1.5\%$
X	-55°C	5	+85°C	C	$\pm 2.2\%$
		6	+105°C	D	$\pm 3.3\%$
		7	+125°C	E	$\pm 4.7\%$
				F	$\pm 7.5\%$
				P	$\pm 10.0\%$
				R	$\pm 15.0\%$
				S	$\pm 22.0\%$
				T	$\pm 22\% \sim 33\%$
				U	$\pm 22\% \sim 56\%$
				V	$\pm 22\% \sim 82\%$

**Example**

X7R: -55°C to +125°C,  $\pm 15.0\%$

## Discrete components

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### Diodes and LEDs

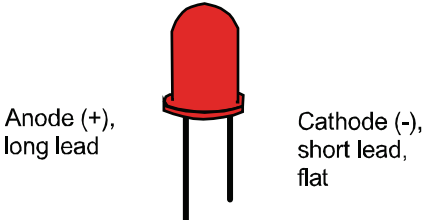
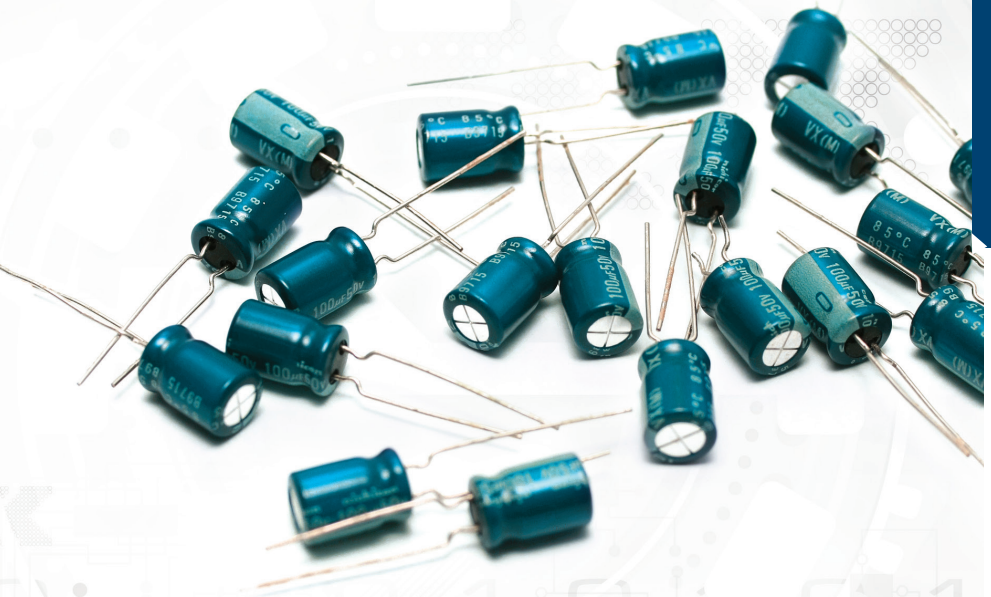


Figure 5: Diode and LED pin names

Color	Voltage
Infrared	1.4
Red	1.7 to 1.9
Orange / yellow	2
Green	2.1
Blue / white	3.4

Figure 6: LED forward voltage drop by color

- Capacitor equations (series, parallel, charge, energy)
- Inductor equations (series, parallel, energy)
- Capacitor charge and discharge
- RMS and mean voltage definition
- RMS for common signals
- Logarithm laws
- dB definitions
- Pole and zero definition with examples



### Capacitor equations

$$C_t = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}} \quad (1) \quad \text{Series capacitors}$$

$$C_t = \frac{C_1 C_2}{C_1 + C_2} \quad (2) \quad \text{Two series capacitors}$$

$$C_t = C_1 + C_2 + \dots + C_N \quad (3) \quad \text{Parallel capacitors}$$

Where

$C_t$  = equivalent total capacitance

$C_1, C_2, C_3 \dots C_N$  = component capacitors

$$Q = CV \quad (4) \quad \text{Charge storage}$$

$$Q = It \quad (5) \quad \text{Charge defined}$$

Where

$Q$  = charge in coulombs (C)

$C$  = capacitance in farads (F)

$I$  = current in amps (A)

$t$  = time in seconds (s)

$$i = C \frac{dv}{dt} \quad (6) \quad \text{Instantaneous current through a capacitor}$$

Where

$i$  = instantaneous current through the capacitor

$C$  = capacitance in farads (F)

$\frac{dv}{dt}$  = the instantaneous rate of voltage change

$$E = \frac{1}{2} CV^2 \quad (7) \quad \text{Energy stored in a capacitor}$$

Where

$E$  = energy stored in an capacitor in Joules (J)

$V$  = voltage in volts

$C$  = capacitance in farads (F)

**Inductor equations**

$$L_t = L_1 + L_2 + \dots + L_N \quad (8) \quad \text{Series inductors}$$

$$L_t = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}} \quad (9) \quad \text{Parallel inductors}$$

$$L_t = \frac{L_1 L_2}{L_1 + L_2} \quad (10) \quad \text{Two parallel inductors}$$

Where

$L_t$  = equivalent total inductance

$L_1, L_2, L_3 \dots L_N$  = component inductance

$$v = L \frac{di}{dt} \quad (11) \quad \text{Instantaneous voltage across an inductor}$$

Where

$v$  = instantaneous voltage across the inductor

$L$  = inductance in Henries (H)

$\frac{d}{dt}$  = the instantaneous rate of voltage change

$$E = \frac{1}{2} LI^2 \quad (12) \quad \text{Energy stored in an Inductor}$$

Where

$E$  = energy stored in an inductor in Joules (J)

$I$  = current in amps

$L$  = inductance in Henries (H)

## Equation for charging a capacitor

$$V_C = V_S \left[ 1 - e^{\left(\frac{-t}{\tau}\right)} \right] \quad (13) \quad \text{General relationship}$$

Where

$V_C$  = voltage across the capacitor at any instant in time ( $t$ )

$V_S$  = the source voltage charging the RC circuit

$t$  = time in seconds

$\tau$  = RC, the time constant for charging and discharging capacitors

Graphing equation 13 produces the capacitor charging curve below. Note that the capacitor is 99.3% charged at five time constants. It is common practice to consider this *fully charged*.

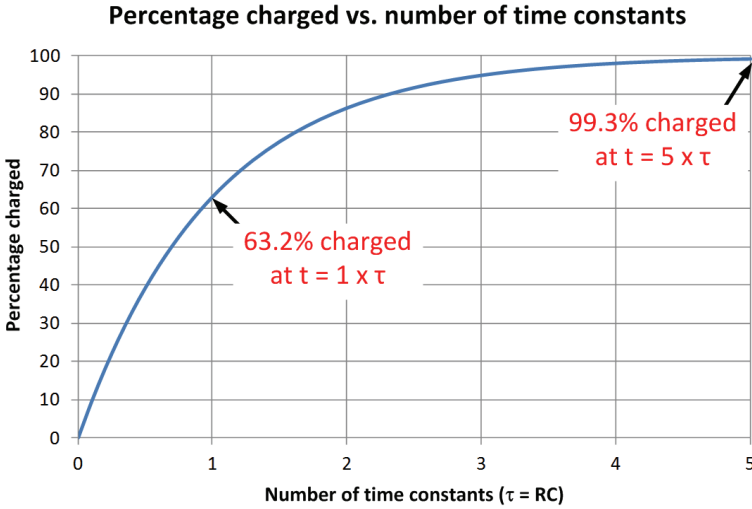


Figure 7: RC charge curve

### Equation for discharging a capacitor

$$V_C = V_i \left[ e^{\left(\frac{-t}{\tau}\right)} \right] \quad (14) \quad \text{General relationship}$$

Where

$V_C$  = voltage across the capacitor at any instant in time (t)

$V_i$  = the initial voltage of the capacitor at  $t=0s$

t = time in seconds

$\tau = RC$ , the time constant for charging and discharging capacitors

Graphing equation 14 produces the capacitor discharge curve below. Note that the capacitor is 0.7% charged at five time constants. It is common practice to consider this *fully discharged*.

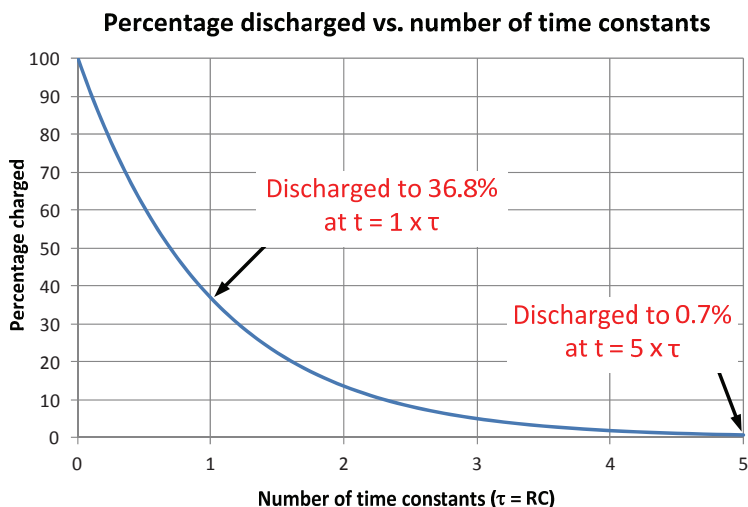


Figure 8: RC discharge curve

## RMS voltage

$$V_{\text{RMS}} = \sqrt{\frac{1}{(T_2 - T_1)} \int_{T_1}^{T_2} [V(t)]^2 dt} \quad (15) \text{ General relationship}$$

Where

$V(t)$  = continuous function of time

$t$  = time in seconds

$T_1 \leq t \leq T_2$  = the time interval that the function is defined over

## Mean voltage

$$V_{\text{MEAN}} = \frac{1}{(T_2 - T_1)} \int_{T_1}^{T_2} V(t) dt \quad (16) \text{ General relationship}$$

Where

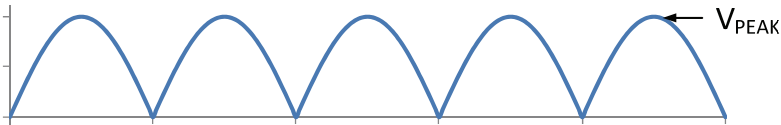
$V(t)$  = continuous function of time

$t$  = time in seconds

$T_1 \leq t \leq T_2$  = the time interval that the function is defined over

$$V_{\text{RMS}} = \frac{V_{\text{PEAK}}}{\sqrt{2}} \quad (17) \text{ RMS for full wave rectified sine wave}$$

$$V_{\text{MEAN}} = \frac{2 \times V_{\text{PEAK}}}{\pi} \quad (18) \text{ Mean for full wave rectified sine wave}$$



**Figure 9: Full wave rectified sine wave**



## RMS voltage and mean voltage

$$V_{\text{RMS}} = V_{\text{PEAK}} \sqrt{\frac{T}{2\tau}} \quad (19) \quad \text{RMS for a half-wave rectified sine wave}$$

$$V_{\text{MEAN}} = \frac{2 \times V_{\text{PEAK}}}{\pi} \left( \frac{T}{\tau} \right) \quad (20) \quad \text{Mean for a half-wave rectified sine wave}$$

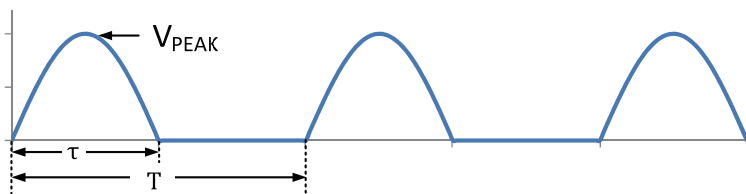


Figure 10: Half-wave rectified sine wave

$$V_{\text{RMS}} = V_{\text{PEAK}} \sqrt{\frac{\tau}{T}} \quad (21) \quad \text{RMS for a square wave}$$

$$V_{\text{MEAN}} = V_{\text{PEAK}} \frac{\tau}{T} \quad (22) \quad \text{Mean for a square wave}$$

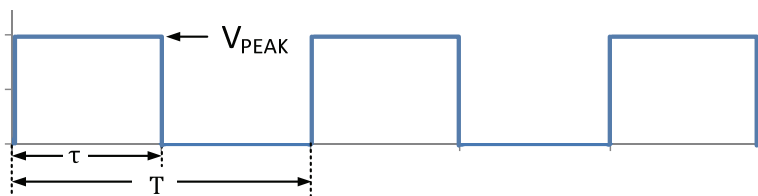


Figure 11: Square wave

## RMS voltage and mean voltage

$$V_{\text{RMS}} = \frac{\tau}{T} \sqrt{\frac{V_a^2 + V_a V_b + V_b^2}{3}} \quad (23) \text{ RMS for a trapezoid}$$

$$V_{\text{MEAN}} = \frac{\tau}{2T} (V_a + V_b) \quad (24) \text{ Mean for a trapezoid}$$

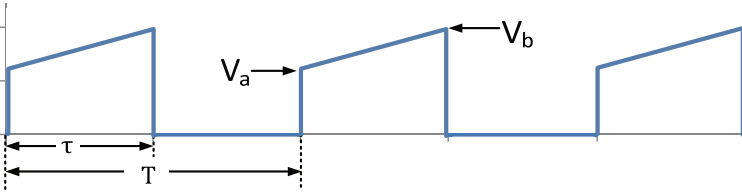


Figure 12: Trapezoidal wave

$$V_{\text{RMS}} = V_{\text{PEAK}} \sqrt{\frac{\tau}{3T}} \quad (25) \text{ RMS for a triangle wave}$$

$$V_{\text{MEAN}} = \frac{\tau}{2T} V_{\text{PEAK}} \quad (26) \text{ Mean for a triangle wave}$$

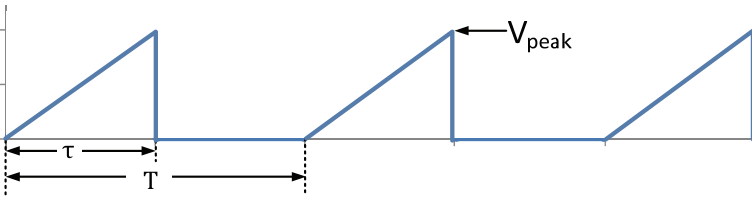


Figure 13: Triangle wave

**Logarithmic mathematical definitions**

$$\log\left(\frac{A}{B}\right) = \log(A) - \log(B) \quad (27) \text{ Log of dividend}$$

$$\log(AB) = \log(A) + \log(B) \quad (28) \text{ Log of product}$$

$$\log(A^x) = x \log(A) \quad (29) \text{ Log of exponent}$$

$$\log_b(X) = \frac{\log_a(X)}{\log_a(b)} \quad (30) \text{ Changing the base of log function}$$

$$\log_2(X) = \frac{\log_{10}(X)}{\log_{10}(2)} \quad (31) \text{ Example changing to log base 2}$$

$$\ln(X) = \log_e(X) \quad (32) \text{ Natural log is log base e}$$

$$e = 2.718282 \quad (33) \text{ Exponential function to 6 digits.}$$

**Alternative notations**

$$\exp(x) = e^x \quad (34) \text{ Different notation for exponential function}$$

$$3.54E - 2 = 3.54 \times 10^{-2} \quad (35) \text{ Different notation for scientific notation, sometimes confused with exponential function}$$

## dB definitions

### Bode plot basics

The frequency response for the magnitude or gain plot is the change in voltage gain as frequency changes. This change is specified on a Bode plot, a plot of frequency versus voltage gain in dB (decibels). Bode plots are usually plotted as semi-log plots with frequency on the x-axis, log scale, and gain on the y-axis, linear scale. The other half of the frequency response is the phase shift versus frequency and is plotted as frequency versus degrees phase shift. Phase plots are usually plotted as semi-log plots with frequency on the x-axis, log scale, and phase shift on the y-axis, linear scale.

### Definitions

$$\text{Voltage gain (dB)} = 20 \log \left( \frac{V_{\text{OUT}}}{V_{\text{IN}}} \right) \quad (36) \text{ Voltage gain in decibels}$$

$$\text{Power gain (dB)} = 10 \log \left( \frac{P_{\text{OUT}}}{P_{\text{IN}}} \right) \quad (37) \text{ Power gain in decibels}$$

$$\text{Power gain (dBm)} = 10 \log \left( \frac{P_{\text{OUT}}}{1 \text{ mW}} \right) \quad (38) \text{ Power gain in decibel milliwatt}$$

**Table 14: Examples of common gain values and dB equivalent**

A (V/V)	A (dB)
0.001	-60
0.01	-40
0.1	-20
1	0
10	20
100	40
1,000	60
10,000	80
100,000	100
1,000,000	120
10,000,000	140

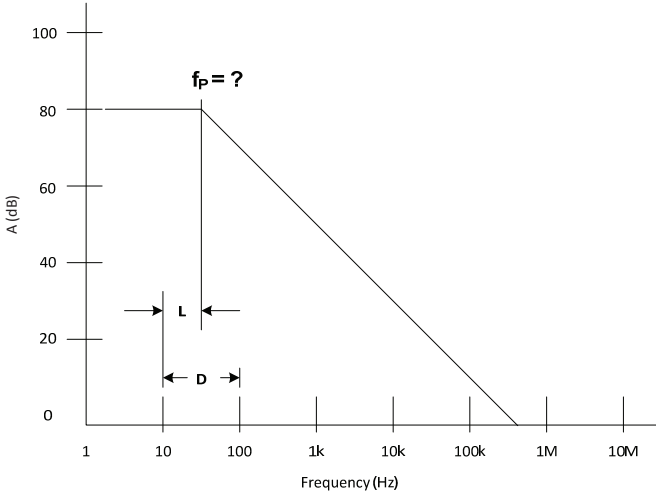
Roll-off rate is the decrease in gain with frequency

Decade is a tenfold increase or decrease in frequency.(from 10 Hz to 100 Hz is one decade)

Octave is the doubling or halving of frequency (from 10 Hz to 20 Hz is one octave)

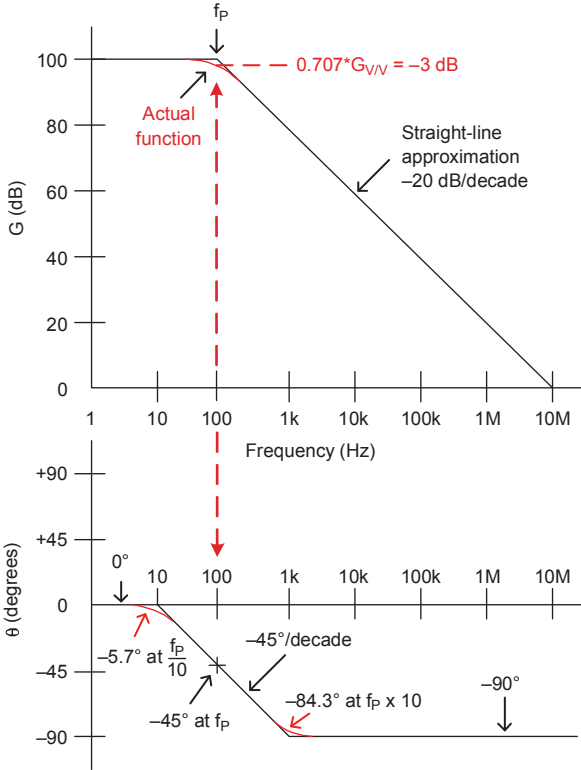
Figure 14 illustrates a method to graphically determine values on a logarithmic axis that are not directly on an axis grid line.

1. Given  $L = 1$  cm;  $D = 2$ cm, measured with a ruler.
2.  $L/D = \log_{10}(f_p)$
3.  $f_p = 10^{(L/D)} = 10^{(1\text{CM}/2\text{CM})} = 3.16$
4. Adjust for the decade range (for example, 31.6 Hz)



**Figure 14: Finding values on logarithmic axis not directly on a grid line**

## Bode plots: Poles



**Figure 15: Pole gain and phase**

- Pole Location =  $f_p$  (cutoff freq)
- Magnitude ( $f < f_p$ ) =  $G_{dc}$  (for example, 100 dB)
- Magnitude ( $f = f_p$ ) =  $-3$  dB
- Magnitude ( $f > f_p$ ) =  $-20$  dB/decade
- Phase ( $f = f_p$ ) =  $-45^\circ$
- Phase ( $0.1 f_p < f < 10 f_p$ ) =  $-45^\circ/\text{decade}$
- Phase ( $f > 10 f_p$ ) =  $-90^\circ$
- Phase ( $f < 0.1 f_p$ ) =  $0^\circ$

**Pole (equations)**

$$G_V = \frac{V_{OUT}}{V_{IN}} = \frac{G_{DC}}{i\left(\frac{f}{f_p}\right) + 1} \quad (39) \text{ As a complex number}$$

$$G_V = \frac{V_{OUT}}{V_{IN}} = \frac{G_{DC}}{\sqrt{\left(\frac{f}{f_p}\right)^2 + 1}} \quad (40) \text{ Magnitude}$$

$$\theta = -\tan^{-1}\left(\frac{f}{f_p}\right) \quad (41) \text{ Phase shift}$$

$$G_{dB} = 20 \text{ Log}(G_V) \quad (42) \text{ Magnitude in dB}$$

Where

$G_V$  = voltage gain in V/V

$G_{dB}$  = voltage gain in decibels

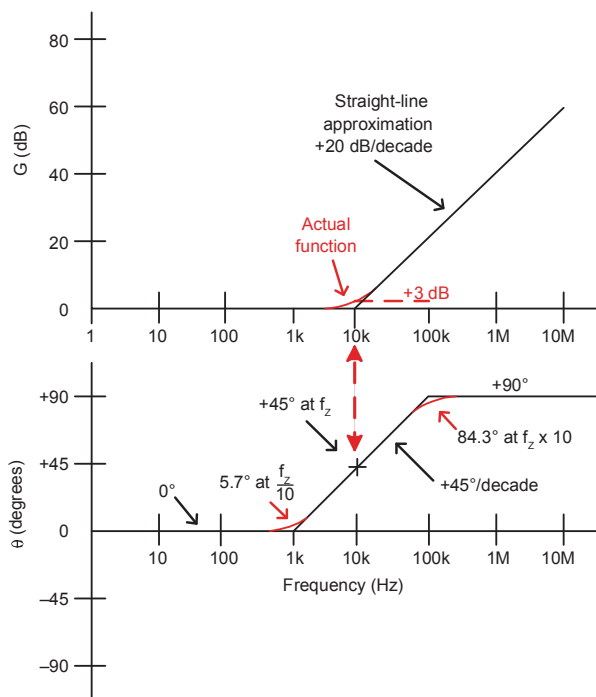
$G_{dc}$  = the dc or low frequency voltage gain

$f$  = frequency in Hz

$f_p$  = frequency at which the pole occurs

$\theta$  = phase shift of the signal from input to output

## Bode plots (zeros)



**Figure 16: Zero gain and phase**

- Zero location =  $f_z$
- Magnitude ( $f < f_z$ ) = 0 dB
- Magnitude ( $f = f_z$ ) = +3 dB
- Magnitude ( $f > f_z$ ) = +20 dB/decade
- Phase ( $f = f_z$ ) =  $+45^\circ$
- Phase ( $0.1 f_z < f < 10 f_z$ ) =  $+45^\circ/\text{decade}$
- Phase ( $f > 10 f_z$ ) =  $+90^\circ$
- Phase ( $f < 0.1 f_z$ ) =  $0^\circ$



**Zero (equations)**

$$G_V = \frac{V_{OUT}}{V_{IN}} = G_{DC} \left[ i \left( \frac{f}{f_Z} \right) + 1 \right] \quad (43) \text{ As a complex number}$$

$$G_V = \frac{V_{OUT}}{V_{IN}} = G_{DC} \sqrt{\left( \frac{f}{f_Z} \right)^2 + 1} \quad (44) \text{ Magnitude}$$

$$\theta = \tan^{-1} \left( \frac{f}{f_Z} \right) \quad (45) \text{ Phase shift}$$

$$G_{dB} = 20 \text{ Log}(G_V) \quad (46) \text{ Magnitude in dB}$$

Where

$G_V$  = voltage gain in V/V

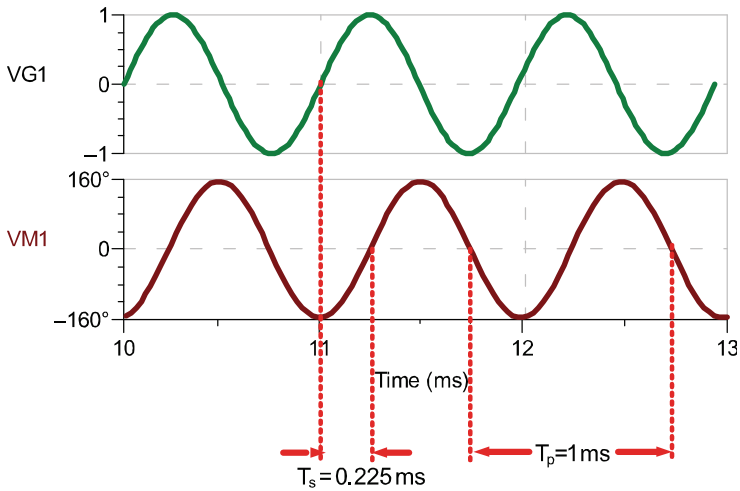
$G_{dB}$  = voltage gain in decibels

$G_{DC}$  = the dc or low frequency voltage gain

$f$  = frequency in Hz

$f_Z$  = frequency at which the zero occurs

$\theta$  = phase shift of the signal from input to output



**Figure 17: Time to phase shift**

$$\theta = \frac{T_S}{T_P} * 360^\circ \quad (47) \quad \text{Phase shift from time}$$

Where

$T_S$  = time shift from input to output signal

$T_P$  = period of signal

$\theta$  = phase shift of the signal from input to output

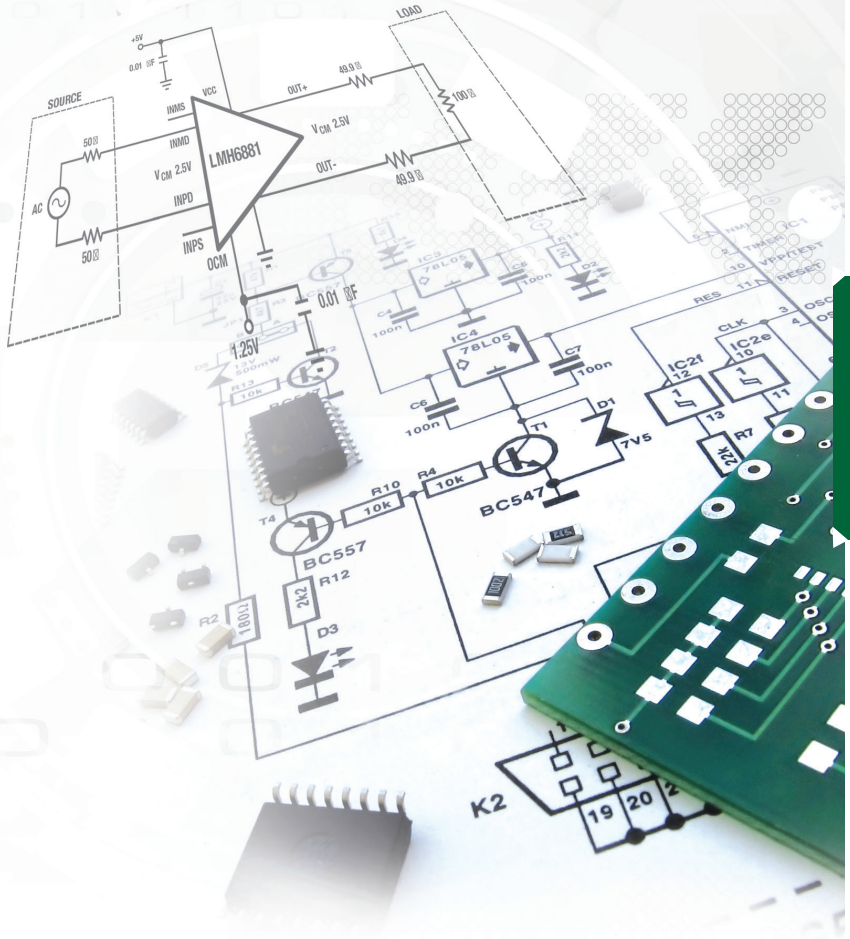
### Example

Calculate the phase shift in degrees for Figure 17.

### Answer

$$\theta = \frac{T_S}{T_P} * 360^\circ = \left( \frac{0.225 \text{ ms}}{1 \text{ ms}} \right) * 360^\circ = 81^\circ$$

- Basic op amp configurations
- Op amp bandwidth
- Full power bandwidth
- Small signal step response
- Noise equations
- Stability equations
- Stability open loop SPICE analysis



# Amplifier

## Basic op amp configurations

$$G_{CL} = 1 \quad (48) \text{ Gain for buffer configuration}$$

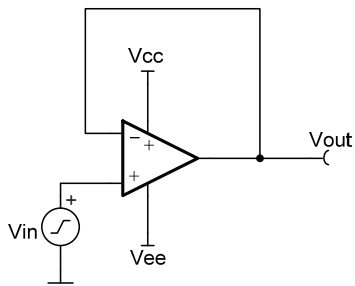


Figure 18: Buffer configuration

$$G_{CL} = \frac{R_f}{R_1} + 1 \quad (49) \text{ Gain for non-inverting configuration}$$

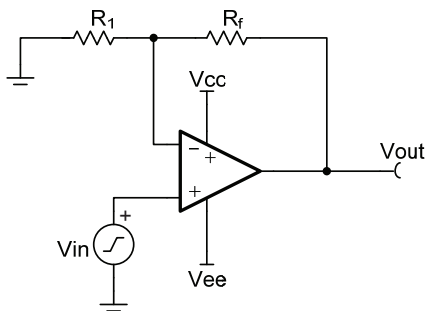
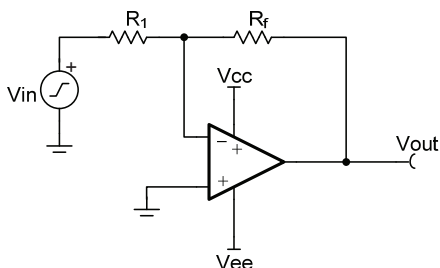


Figure 19: Non-inverting configuration

**Basic op amp configurations (cont.)**

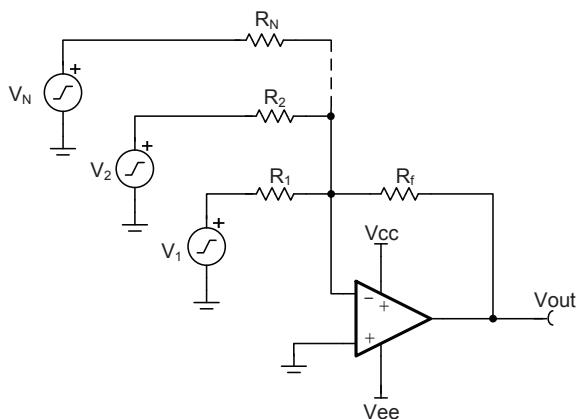
$$G_{CL} = -\frac{R_f}{R_1} \quad (50) \quad \text{Gain for inverting configuration}$$



**Figure 20: Inverting configuration**

$$V_{OUT} = -R_f \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_N}{R_N} \right) \quad (51) \quad \text{Transfer function for inverting summing amplifier}$$

$$V_{OUT} = -\frac{R_f}{R_1} (V_1 + V_2 + \dots + V_N) \quad (52) \quad \text{Transfer function for inverting summing amplifier, assuming } R_1 = R_2 = \dots = R_N$$



**Figure 21: Inverting summing configuration**

# Amplifier

## Basic op amp configurations (cont.)

$$V_{OUT} = \left( \frac{R_f}{R_{in}} + 1 \right) \left[ \frac{V_1}{N} + \frac{V_2}{N} + \dots + \frac{V_N}{N} \right]$$

(53) Transfer function for non-inverting summing amplifier for equal input resistors

Where

$R_1 = R_2 = \dots = R_N$

N = number of input resistors

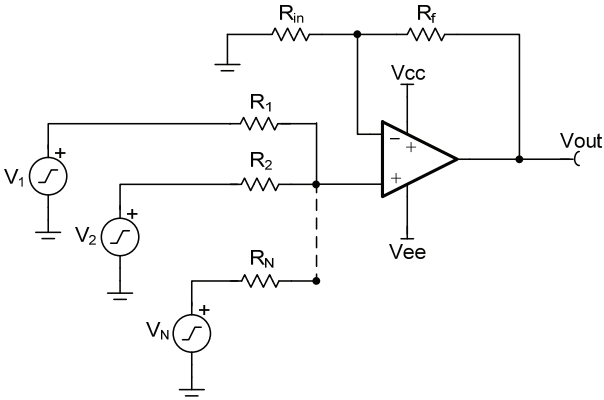


Figure 22: Non-inverting summing configuration

Simple non-inverting amp with  $C_f$  filter

$G_{LF} = \frac{R_f}{R_1} + 1$  (54) Gain for non-inverting configuration for  $f < f_c$

$G_{HF} = 1$  (55) Gain for non-inverting configuration for  $f \gg f_c$

$f_c = \frac{1}{2\pi R_f C_f}$  (56) Cut off frequency for non-inverting configuration

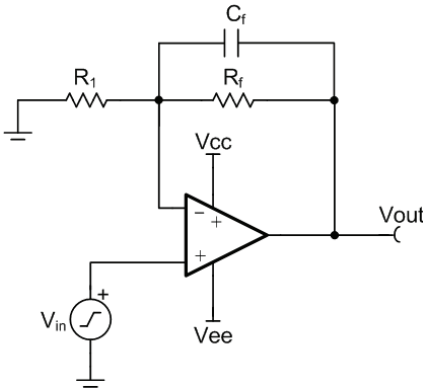


Figure 23: Non-inverting amplifier with  $C_f$  filter

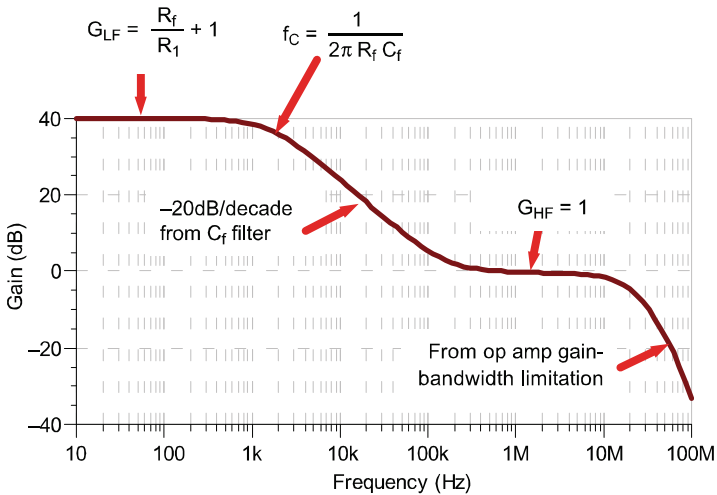


Figure 24: Frequency response for non-inverting op amp with  $C_f$  filter

# Amplifier

## Simple inverting amp with $C_f$ filter

$$G_{LF} = -\frac{R_f}{R_1} \quad (57) \quad \text{Gain for inverting configuration for } f < f_c$$

$$G_{HF} = 1 \quad (58) \quad \text{Gain for inverting configuration for } f \gg f_c$$

$$f_c = \frac{1}{2\pi R_f C_f} \quad (59) \quad \text{Cutoff frequency for inverting configuration}$$

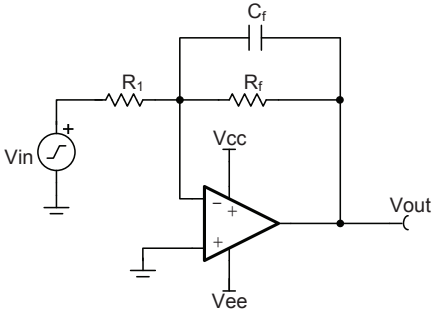


Figure 25: Inverting amplifier with  $C_f$  filter

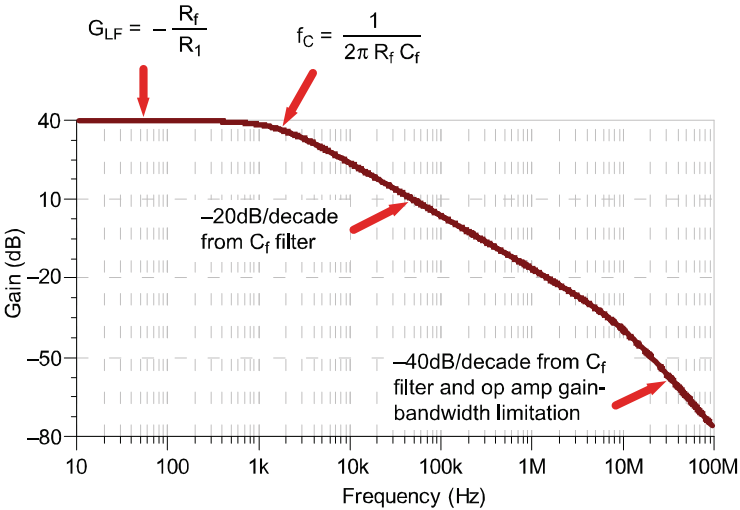


Figure 26: Frequency response for inverting op amp with  $C_f$  filter



**Op amp bandwidth**

$$GBW = \text{Gain} \times BW \quad (60) \quad \text{Gain bandwidth product defined}$$

Where

GBW = gain bandwidth product, listed in op amp data sheet specification table

Gain = closed loop gain, set by op amp gain configuration

BW = the bandwidth limitation of the amplifier

**Example**

Determine bandwidth using equation 60

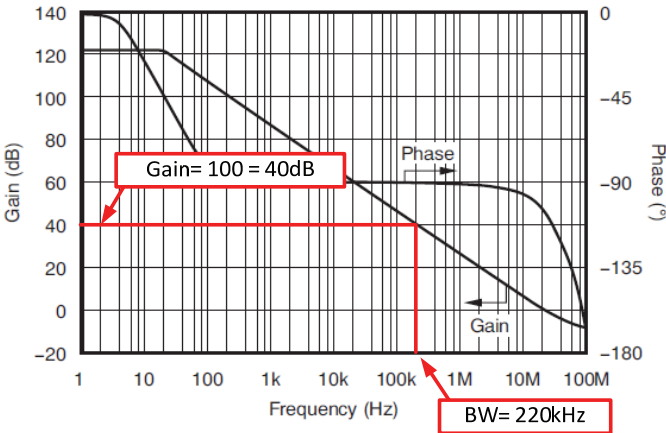
$$\text{Gain} = 100 \quad (\text{from amplifier configuration})$$

$$GBW = 22\text{MHz} \quad (\text{from data sheet})$$

$$BW = \frac{GBW}{\text{Gain}} = \frac{22\text{MHz}}{100} = 220\text{kHz}$$

Note that the same result can be graphically determined using the  $A_{OL}$  curve as shown below.

**Open-loop gain and phase vs. frequency**



**Figure 27: Using  $A_{OL}$  to find closed-loop bandwidth**

## Full power bandwidth

$$V_P = \frac{SR}{2\pi f} \quad (61) \quad \text{Maximum output without slew-rate induced distortion}$$

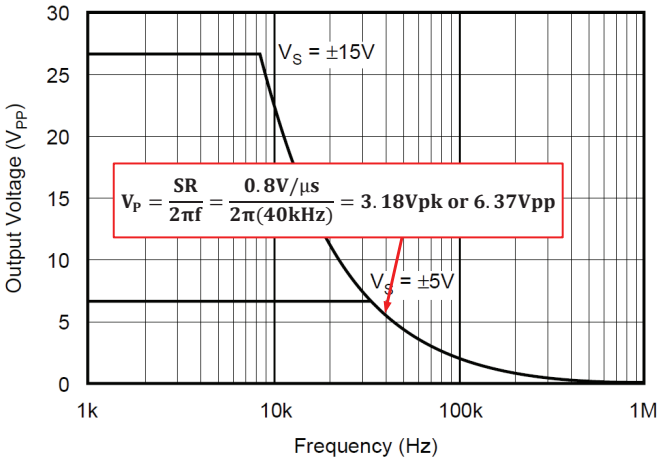
Where

$V_P$  = maximum peak output voltage before slew induced distortion occurs

SR = slew rate

$f$  = frequency of applied signal

## Maximum output voltage vs. frequency



**Figure 28: Maximum output without slew-rate induced distortion**

Notice that the above figure is graphed using equation 61 for the OPA188. The example calculation shows the peak voltage for the OPA277 at 40kHz. This can be determined graphically or with the equation.

## Example

$$V_P = \frac{SR}{2\pi f} = \frac{0.8V/\mu s}{2\pi(40kHz)} = 3.18V_{pk} \text{ or } 6.37V_{pp}$$

**Small signal step response**

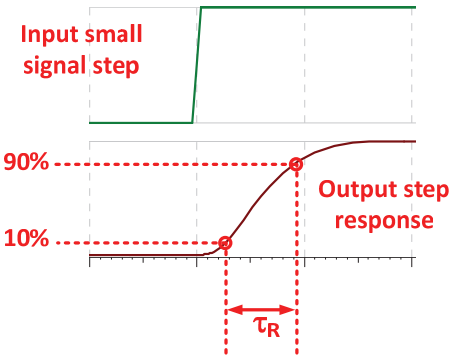
$$\tau_R = \frac{0.35}{f_C} \quad (62) \quad \text{Rise time for a small signal step}$$

Where

$\tau_R$  = the rise time of a small signal step response

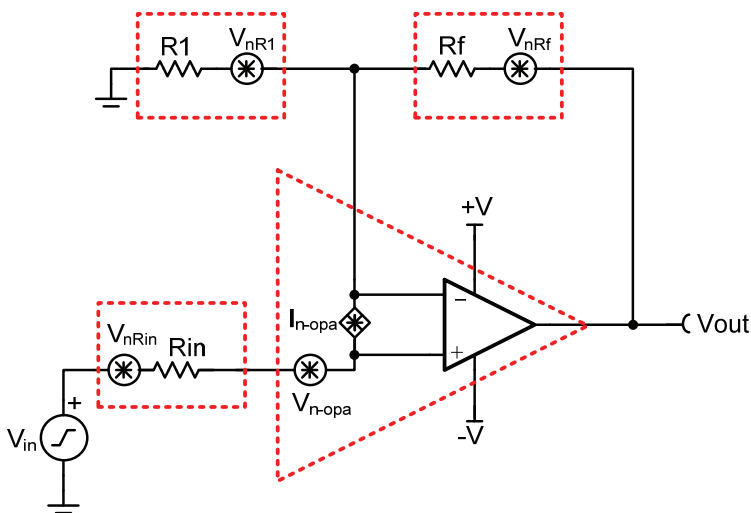
$f_C$  = the closed-loop bandwidth of the op amp circuit

**Small signal step response waveform**



**Figure 29: Small signal step response**

## Op amp noise model



**Figure 30: Op amp noise model**

Op amp intrinsic noise includes:

- Noise caused by op amp (current noise + voltage noise)
- Resistor noise

**Noise bandwidth calculation**

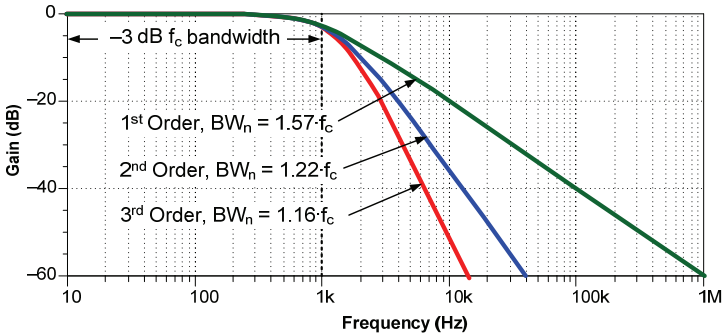
$$BW_N = K_N f_c \quad (63) \text{ Noise bandwidth}$$

Where

$BW_N$  = noise bandwidth of the system

$K_N$  = the brick wall correction factor for different filter order

$f_c$  = -3 dB bandwidth of the system



**Figure 31: Op amp bandwidth for three different filters orders**

**Table 15: Brick wall correction factors for noise bandwidth**

Number of poles	$K_N$ brick wall correction factor
1	1.57
2	1.22
3	1.13
4	1.12

**Broadband total noise calculation**

$$E_N = e_{BB} \sqrt{BW_N} \quad (64) \text{ Total rms noise from broadband}$$

Where

$E_N$  = total rms noise from broadband noise

$e_{BB}$  = broadband noise spectral density (nV/rtHz)

$BW_N$  = noise bandwidth (Hz)

## 1/f total noise calculation

$$E_{N\_NORMAL} = e_{BF}\sqrt{f_O} \quad (65) \quad \text{Normalized 1/f noise at 1 Hz}$$

Where

$E_{N\_NORMAL}$  = 1/f noise normalized to 1 Hz

$e_{BF}$  = noise spectral density measured in the 1/f region

$f_O$  = the frequency that the 1/f noise  $e_{BF}$  is measured at

$$E_{N\_FLICKER} = E_{N\_NORMAL} \sqrt{\ln\left(\frac{f_H}{f_L}\right)} \quad (66) \quad \text{1/f total noise calculation}$$

Where

$E_{N\_FLICKER}$  = total rms noise from flicker

$E_{N\_NORMAL}$  = 1/f noise normalized to 1 Hz

$f_H$  = upper cutoff frequency or noise bandwidth

$f_L$  = lower cutoff frequency, normally set to 0.1 Hz

**Table 16: Peak-to-peak conversion**

Number of standard deviations	Percent chance reading is in range
2 $\sigma$ (same as $\pm 1\sigma$ )	68.3%
3 $\sigma$ (same as $\pm 1.5\sigma$ )	86.6%
4 $\sigma$ (same as $\pm 2\sigma$ )	95.4%
5 $\sigma$ (same as $\pm 2.5\sigma$ )	98.8%
6 $\sigma$ (same as $\pm 3\sigma$ )	99.7%
6.6 $\sigma$ (same as $\pm 3.3\sigma$ )	99.9%

**Thermal noise calculation**

$$E_{N,R} = \sqrt{4 kTR\Delta f} \quad (67) \quad \text{Total rms thermal noise}$$

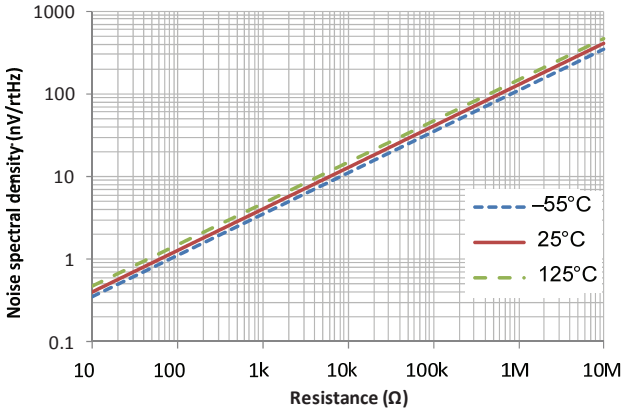
Where

$E_{N,R}$  = total rms noise from resistance, also called thermal noise

$k$  = Boltzmann's constant  $1.38 \times 10^{-23}$  J/K

$T$  = temperature in Kelvin

$\Delta f$  = noise bandwidth in Hz



**Figure 32: Noise spectral density vs. resistance**

## Ac response versus frequency

Figure 33 illustrates a bode plot with four different examples of ac peaking.

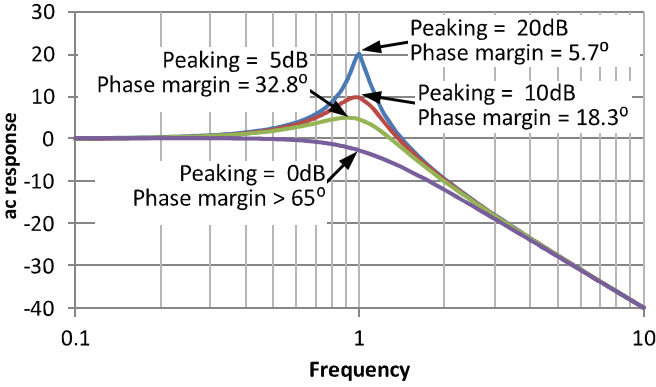


Figure 33: Stability – ac peaking relationship example

## Phase margin versus ac peaking

This graph illustrates the phase margin for any given level of ac peaking. Note that 45° of phase margin or greater is required for stable operation.

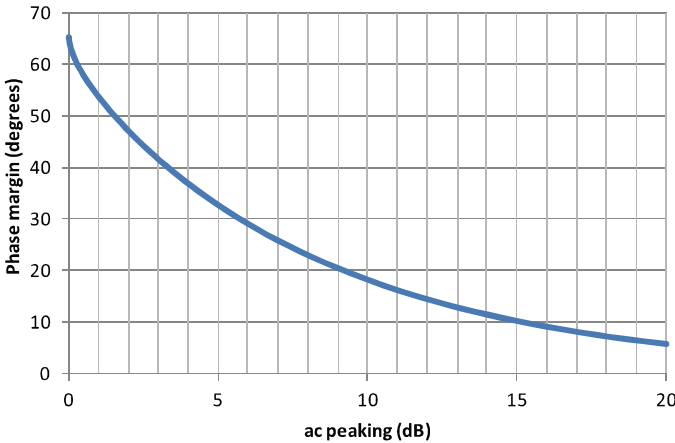
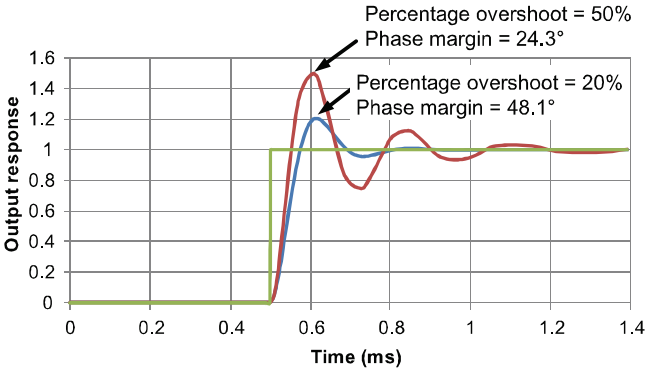


Figure 34: Stability – phase margin vs. peaking for a two-pole system



**Transient overshoot**

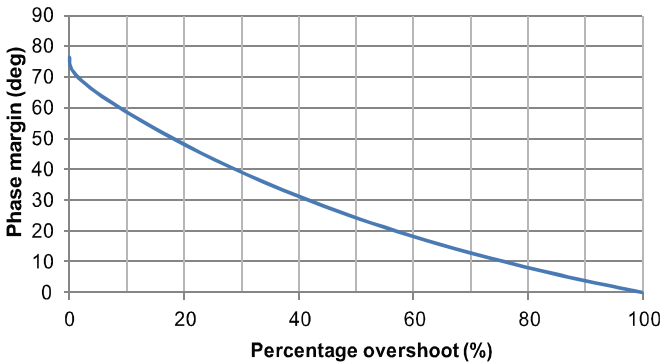
Figure 35 illustrates a transient response with two different examples of percentage overshoot.



**Figure 35: Stability – transient overshoot example**

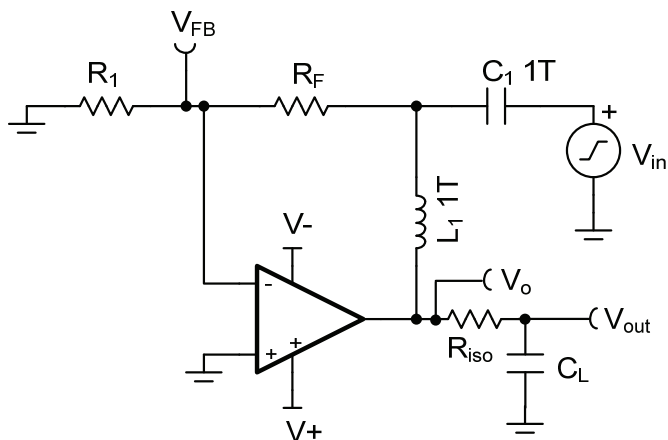
**Phase margin versus percentage overshoot**

This graph illustrates the phase margin for any given level of transient overshoot. Note that 45° of phase margin or greater is required for stable operation.



**Figure 36: Stability – phase margin vs. percentage overshoot**

Note: The curves assume a two-pole system.



**Figure 37: Common spice test circuit used for stability**

$$A_{OL\_LOADED} = \frac{V_o}{V_{FB}} \quad (68) \quad \text{Loaded open-loop gain}$$

$$\beta = V_{FB} \quad (69) \quad \text{Feedback factor}$$

$$\frac{1}{\beta} = \frac{1}{V_{FB}} \quad (70) \quad \text{Closed-loop noise gain}$$

$$A_{OL\_LOADED} \times \beta = V_o \quad (71) \quad \text{Loop gain}$$

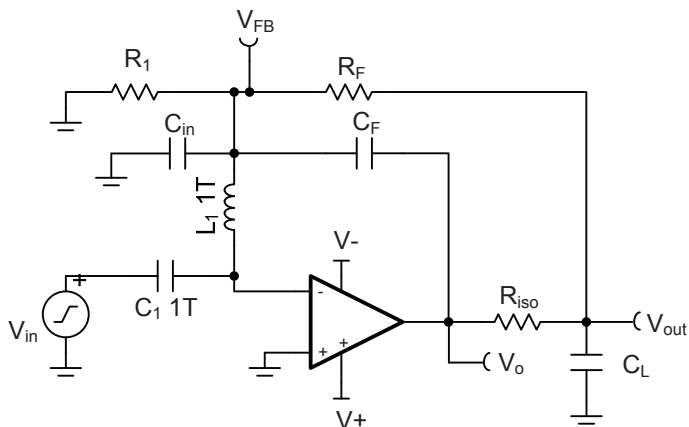
Where

$V_o$  = the voltage at the output of the op amp.

$V_{OUT}$  = the voltage output delivered to the load, which may be important to the application but is not considered in stability analysis.

$V_{FB}$  = feedback voltage

$R_F$ ,  $R_1$ ,  $R_{iso}$  and  $C_L$  = the op amp feedback network and load. Other op amp topologies will have different feedback networks; however, the test circuit will be the same for most cases. Figure 38 shows the exception to the rule (multiple feedback).  $C_1$  and  $L_1$  are components that facilitate SPICE analysis. They are large (1TF, 1TH) to make the circuit closed-loop for dc, but open loop for ac frequencies. SPICE requires closed-loop operation at dc for convergence.



**Figure 38: Alternative (multiple feedback) SPICE test circuit used for stability**

$$A_{OL\_LOADED} = V_O \quad (72) \quad \text{Loaded open loop gain}$$

$$\beta = \frac{V_{FB}}{V_O} \quad (73) \quad \text{Feedback factor}$$

$$\frac{1}{\beta} = \frac{V_O}{V_{FB}} \quad (74) \quad \text{Closed-loop noise gain}$$

$$A_{OL\_LOADED} \times \beta = V_{FB} \quad (75) \quad \text{Loop gain}$$

Where

$V_O$  = the voltage at the output of the op amp.

$V_{OUT}$  = the voltage output delivered to the load. This may be important to the application but is not considered in stability analysis.

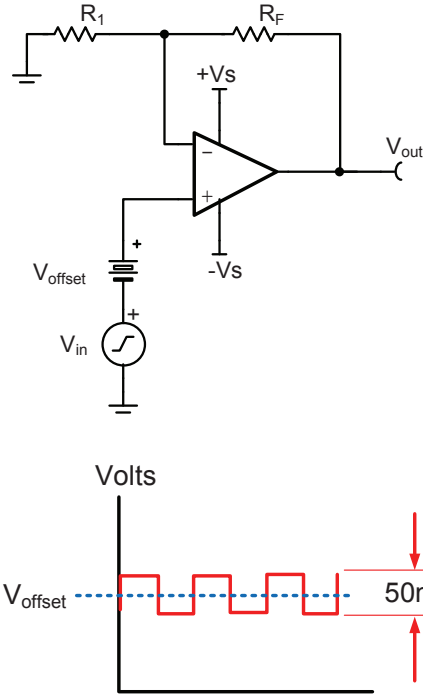
$V_{FB}$  = feedback voltage

$R_F$ ,  $R_1$ ,  $R_{ISO}$  and  $C_F$  = the op amp feedback network. Because there are two paths for feedback, the loop is broken at the input.

$C_1$  and  $L_1$  are components that facilitate SPICE analysis. They are large (1TF, 1TH) to make the circuit closed loop for dc, but open loop for ac frequencies. SPICE requires closed-loop operation at dc for convergence.

$C_{IN}$  = the equivalent input capacitance taken from the op amp datasheet. This capacitance normally does not need to be added because the model includes it.

However, when using this simulation method the capacitance is isolated by the 1TH inductor.



**Figure 39: Transient real world stability test**

### Test tips

- Choose test frequency  $\ll f_{cl}$
- Small signal ( $V_{pp} \leq 50\text{ mV}$ ) ac output square wave (for example, 1 kHz)
- Adjust  $V_{in}$  amplitude to yield output  $\leq 50\text{ mVpp}$
- Worst cases is usually when  $V_{offset} = 0$  (Largest  $R_o$ , for  $I_{out} = 0\text{A}$ ).
- User  $V_{offset}$  as desired to check all output operating points for stability
- Set scope = ac couple and expand vertical scope scale to look for amount of overshoot, undershoot, and ringing on  $V_{out}$
- Use 1x attenuation scope probe on  $V_{out}$  for best resolution

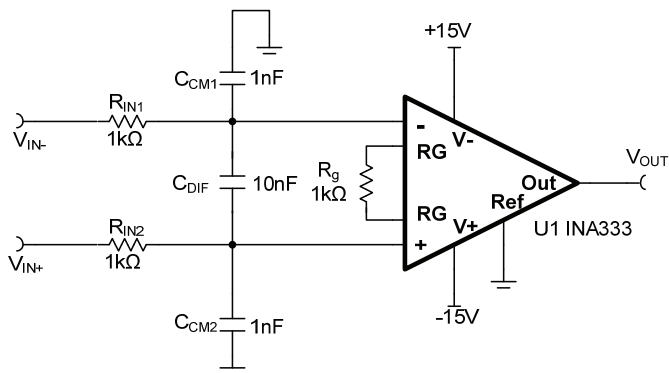


Figure 40: Input filter for instrumentation amplifier

Select  $C_{DIF} \geq 10C_{CM1}$

(76) Differential filter is sized 10 times the common-mode filter

$R_{IN1} = R_{IN2}$

(77) Input resistors must be equal

$C_{CM1} = C_{CM2}$

(78) Common-mode capacitors must be equal

$$f_{CM} = \frac{1}{2\pi R_{IN1} C_{CM1}}$$

(79) Differential filter cutoff

$$f_{DIF} = \frac{1}{2\pi(2R_{IN1})(C_{DIF} + \frac{1}{2}C_{CM1})}$$

(80) Common-mode filter cutoff

Where

$f_{DIF}$  = differential cutoff frequency

$f_{CM}$  = common-mode cutoff frequency

$R_{IN}$  = input resistance

$C_{CM}$  = common-mode filter capacitance

$C_{DIF}$  = differential filter capacitance

Note: Selecting  $C_{DIF} \geq 10 C_{CM}$  sets the differential mode cutoff frequency 10 times lower than the common-mode cutoff frequency. This prevents common-mode noise from being converted into differential noise due to component tolerances.

# Amplifier

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Notes

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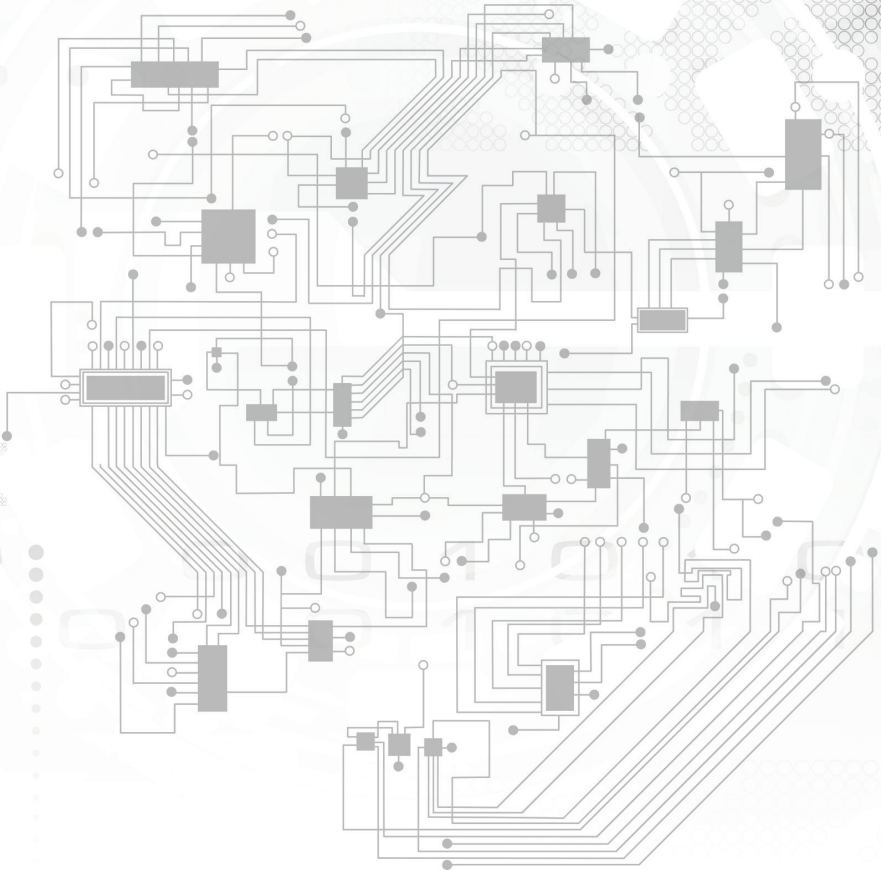
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## PCB and wire

- PCB trace resistance for 1oz and 2oz Cu
- Conductor spacing in a PCB for safe operation
- Current carrying capacity of copper conductors
- Package types and dimensions
- PCB trace capacitance and inductance
- PCB via capacitance and inductance
- Common coaxial cable specifications
- Coaxial cable equations
- Resistance per length for wire types
- Maximum current for wire types



**Table 17: Printed circuit board conductor spacing**

Voltage between conductors (dc or ac peaks)	Minimum spacing						
	Bare board				Assembly		
	B1	B2	B3	B4	A5	A6	A7
0-15	0.05 mm [0.00197 in]	0.1 mm [0.0039 in]	0.1 mm [0.0039 in]	0.05 mm [0.00197 in]	0.13 mm [0.00512 in]	0.13 mm [0.00512 in]	0.13 mm [0.00512 in]
16-30	0.05 mm [0.00197 in]	0.1 mm [0.0039 in]	0.1 mm [0.0039 in]	0.05 mm [0.00197 in]	0.13 mm [0.00512 in]	0.25 mm [0.00984 in]	0.13 mm [0.00512 in]
31-50	0.1 mm [0.0039 in]	0.6 mm [0.024 in]	0.6 mm [0.024 in]	0.13 mm [0.00512 in]	0.13 mm [0.00512 in]	0.4 mm [0.016 in]	0.13 mm [0.00512 in]
51-100	0.1 mm [0.0039 in]	0.6 mm [0.024 in]	1.5 mm [0.0591 in]	0.13 mm [0.00512 in]	0.13 mm [0.00512 in]	0.5 mm [0.020 in]	0.13 mm [0.00512 in]
101-150	0.2 mm [0.0079 in]	0.6 mm [0.024 in]	3.2 mm [0.126 in]	0.4 mm [0.016 in]	0.4 mm [0.016 in]	0.8 mm [0.031 in]	0.4 mm [0.016 in]
151-170	0.2 mm [0.0079 in]	1.25 mm [0.0492 in]	3.2 mm [0.126 in]	0.4 mm [0.016 in]	0.4 mm [0.016 in]	0.8 mm [0.031 in]	0.4 mm [0.016 in]
171-250	0.2 mm [0.0079 in]	1.25 mm [0.0492 in]	6.4 mm [0.252 in]	0.4 mm [0.016 in]	0.4 mm [0.016 in]	0.8 mm [0.031 in]	0.4 mm [0.016 in]
251-300	0.2 mm [0.0079 in]	1.25 mm [0.0492 in]	12.5 mm [0.492 in]	0.4 mm [0.016 in]	0.4 mm [0.016 in]	0.8 mm [0.031 in]	0.8 mm [0.031 in]
301-500	0.25 mm [0.00984 in]	2.5 mm [0.0984 in]	12.5 mm [0.492 in]	0.8 mm [0.031 in]	0.8 mm [0.031 in]	1.5 mm [0.0591 in]	0.8 mm [0.031 in]

B1 Internal conductors

B2 External conductors uncoated sea level to 3050 m

B3 External conductors uncoated above 3050 m

B4 External conductors coated with permanent polymer coating (any elevation)

A5 External conductors with conformal coating over assembly (any elevation)

A6 External component lead/termination, uncoated, sea level to 3050 m

A7 External component lead termination, with conformal coating (any elevation)

Extracted with permission from IPC-2221B, Table 6-1.

For additional information, the entire specification can be downloaded at [www.ipc.org](http://www.ipc.org)



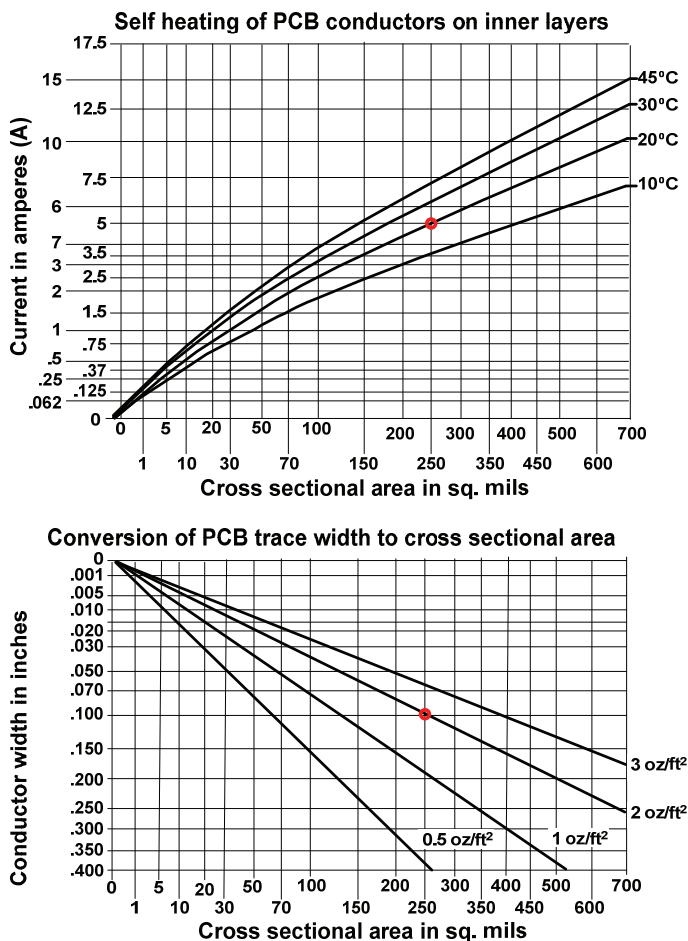


Figure 41: Self heating of PCB traces on inside layer

#### Example

Find the current that will cause a 20°C temperature rise in a PCB trace that is 0.1 inch wide and uses 2 oz/ft<sup>2</sup> copper. (Assume traces on outside of PCB.)

#### Answer

First translate 0.1 inch to 250 sq. mils. using bottom chart. Next find the current associated with 10°C and 250 sq. mils. using top chart (Answer = 5A).

Extracted with permission from IPC-2152, Figure 5-1.

For additional information the entire specification can be downloaded at [www.ipc.org](http://www.ipc.org)

## PCB and wire

### PCB trace resistance for 1 oz Cu

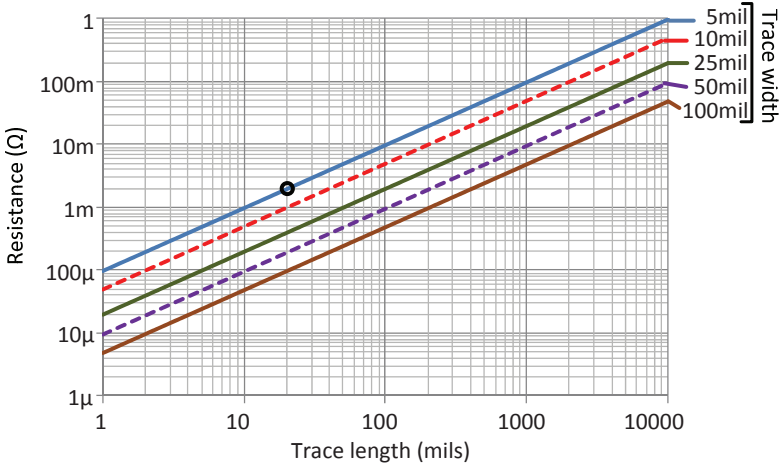


Figure 42: PCB trace resistance vs. length and width for 1 oz-Cu, 25°C

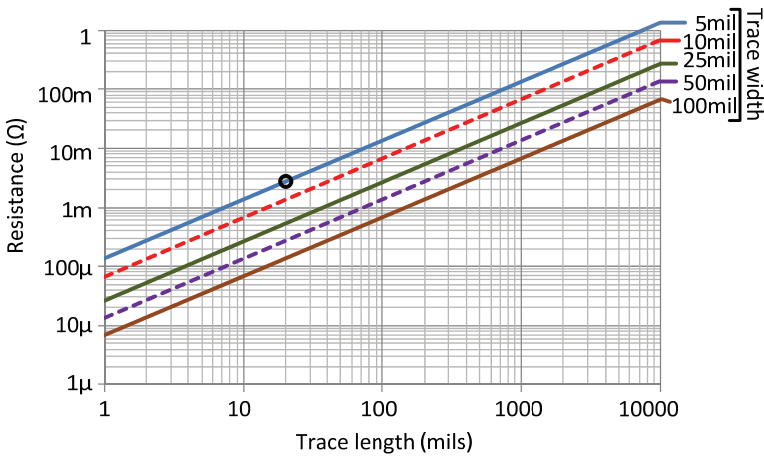


Figure 43: PCB trace resistance vs. length and width for 1 oz-Cu, 125°C

### Example

What is the resistance of a 20 mil long, 5 mil wide trace for a 1 oz-Cu thickness at 25°C and 125°C?

### Answer

$R_{25C} = 2\text{ m}\Omega$ ,  $R_{125C} = 3\text{ m}\Omega$ . The points are circled on the curves.

PCB trace resistance for 2 oz Cu

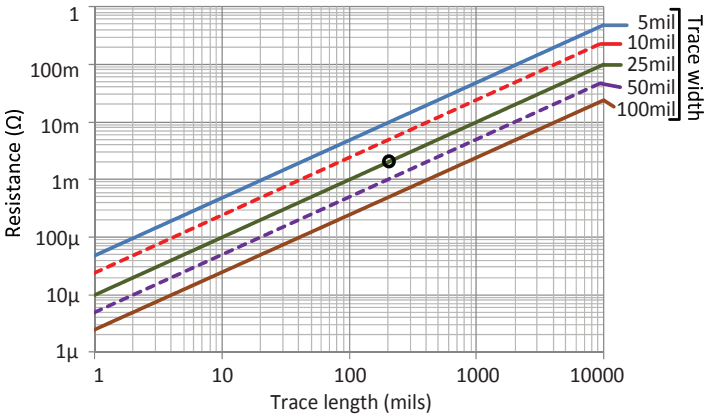


Figure 44: PCB trace resistance vs. length and width for 2 oz-Cu, 25°C

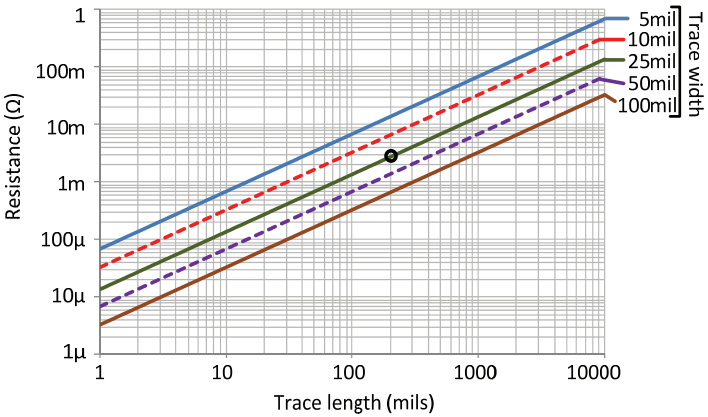


Figure 45: PCB trace resistance vs. length and width for 2 oz-Cu, 125°C

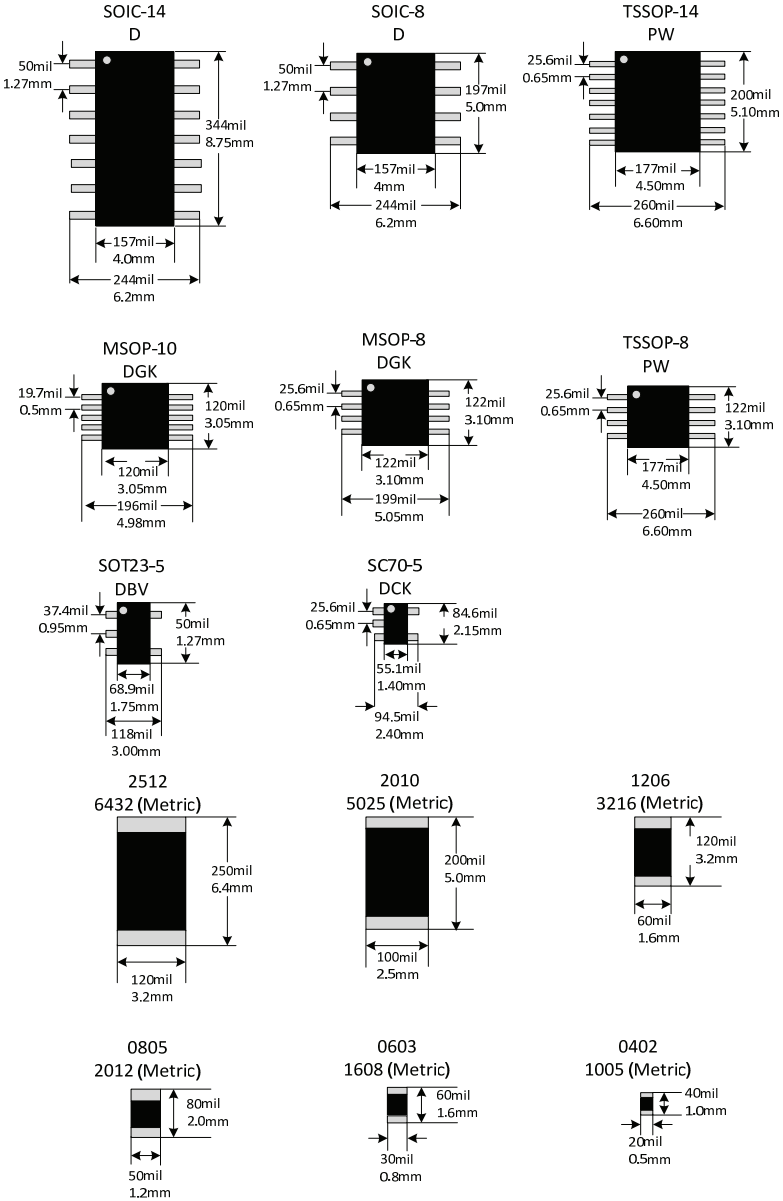
Example

What is the resistance of a 200 mil long, 25 mil wide trace for a 2 oz-Cu thickness at 25°C and 125°C?

Answer

R<sub>25C</sub> = 2 mΩ, R<sub>125C</sub> = 3 mΩ. The points are circled on the curves.

## Common package type and dimensions



**PCB parallel plate capacitance**

$$C(\text{pF}) = \frac{2.249 * 10^{-4} * \epsilon_r * l * w}{h} \quad (81) \quad \text{Capacitance for parallel copper planes}$$

Where

$\epsilon_r$  = PCB dielectric constant ( $\epsilon_r \approx 4.2$  for FR-4)

$l$  = common length of copper planes (mils)

$w$  = common width of copper planes (mils)

$h$  = separation between copper planes (mils)

**Example**

$\epsilon_r = 4.2$

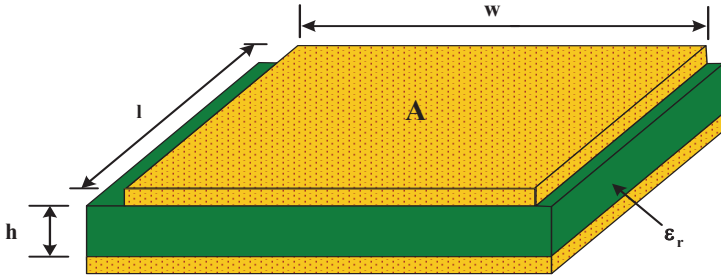
$l = 400$  mils

$w = 400$  mils

$h = 63$  mils

**Answer**

$C = 2.4$  pF



**Figure 46: PCB parallel plate capacitance**

## PCB and wire

### PCB via capacitance and inductance

$$L(\text{nH}) \approx \frac{h}{197} \left[ 1 + \ln \left( \frac{4 * h}{d} \right) \right] \quad (82) \quad \text{Inductance for via}$$

$$C(\text{pF}) \approx \frac{1.41 * 10^{-3} * \epsilon_r * h * d_1}{d_2 - d_1} \quad (83) \quad \text{Capacitance for via}$$

Where

$h$  = separation between planes (mils)

$d$  = diameter of via hole (mils)

$\epsilon_r$  = PCB dielectric constant ( $\epsilon_r \approx 4.2$  for FR-4)

$d_1$  = diameter of the pad surrounding the via (mils)

$d_2$  = diameter of the clearance hole in the plane (mils)

### Example

$h = 63$  mils

$d = 16$  mils

$\epsilon_r = 4.2$

$d_1 = 32$  mils

$d_2 = 63$  mils

### Answer

$L = 1.20$  nH

$C = 0.39$  pF

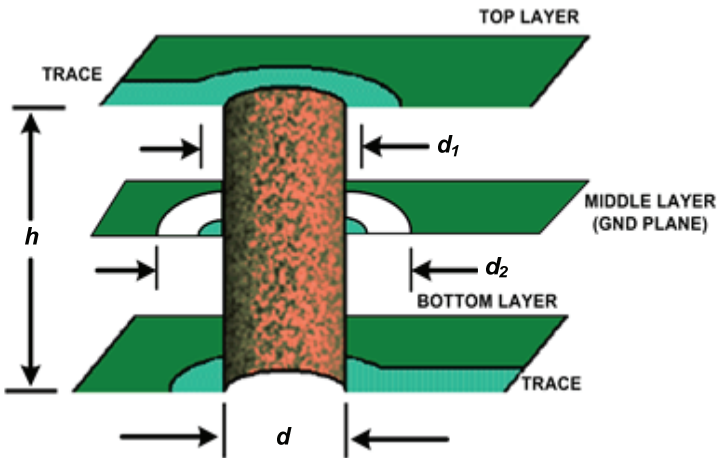


Figure 47: Inductance and capacitance of via

**Adjacent copper trace capacitance**

$$C(\text{pF}) \approx \frac{2.249 * 10^{-4} * l * t}{d} \quad (58) \quad \text{Same layer}$$

$$C(\text{pF}) \approx \frac{2.249 * 10^{-4} * \epsilon_r * l * w}{h} \quad (59) \quad \text{Different layers}$$

Where

$l$  = length of copper trace (mils)

$t$  = thickness of copper trace (mils)

copper thickness (mils) = 1.37 \* (number of ounces)

ex: 1 oz. copper thickness = 1.37 mils

ex: ½ oz. copper thickness = 0.685 mils

$d$  = distance between traces (mils)

$\epsilon_r$  = PCB dielectric constant ( $\epsilon_r \approx 4.2$  for FR-4)

$w$  = width of copper trace (mils)

$h$  = separation between planes (mils)

**Example**

$l$  = 100 mils

$t$  = 1.37 mils (1 oz. copper)

$d$  = 10 mils

$\epsilon_r$  = 4.2

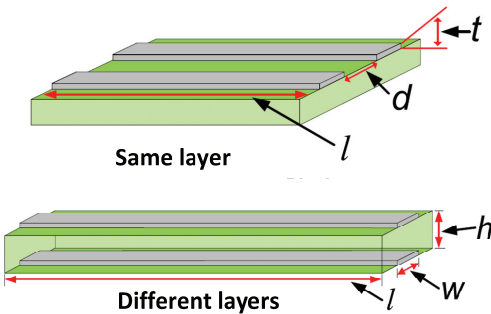
$w$  = 25 mils

$h$  = 63 mils

**Answer**

$C$  (same layer) = 0.003 pF

$C$  (different layers) = 0.037 pF



**Figure 48: Capacitance for adjacent copper traces**

Table 18: Coaxial cable information

Type	$Z_0$	Capacitance / length (pF/feet)	Outside diameter (inches)	dB attenuation /100 ft at 750 MHz	Dielectric type	Application
RG-58	53.5 $\Omega$	28.8	0.195	13.1	PE	Test equipment and RF power to a few hundred watts, and a couple hundred MHz
RG-8	52 $\Omega$	29.6	0.405	5.96	PE	RF power to a few kW, up to several hundred MHz
RG-214/U	50 $\Omega$	30.8	0.425	6.7	PE	
9914	50 $\Omega$	26.0	0.405	4.0	PE	
RG-6	75 $\Omega$	20	0.270	5.6	PF	Video and CATV applications. RF to a few hundred watts, up to a few hundred MHz, sometimes to higher frequencies if losses can be tolerated
RG-59/U	73 $\Omega$	29	0.242	9.7	PE	
RG-11/U	75 $\Omega$	17	0.412	3.65	PE	RF power to a few kW, up to several hundred MHz
RG-62/U	93 $\Omega$	13.5	0.242	7.1	ASP	Used in some test equipment and 100 $\Omega$ video applications
RG-174	50 $\Omega$	31	0.100	23.5	PE	Miniature coax used primarily for test equipment interconnection. Usually short runs due to higher loss.
RG-178/U	50 $\Omega$	29	0.071	42.7	ST	



**Coaxial cable equations**

$$\frac{C}{l} = \frac{2\pi\epsilon}{\ln\left(\frac{D}{d}\right)} \quad (84) \quad \text{Capacitance per length}$$

$$\frac{L}{l} = \frac{\mu}{2\pi} \ln\left(\frac{D}{d}\right) \quad (85) \quad \text{Inductance per length}$$

$$Z_o = \sqrt{\frac{L}{C}} = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \quad (86) \quad \text{Characteristic impedance}$$

Where

L = inductance in henries (H)

C = capacitance in farads (F)

Z = impedance in ohms ( $\Omega$ )

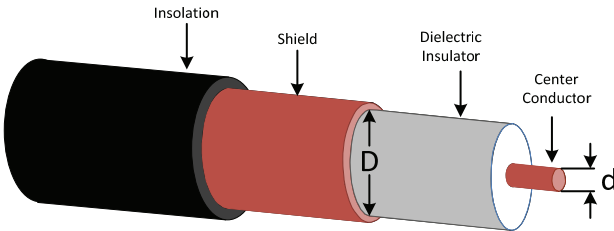
d = diameter of inner conductor

D = inside diameter of shield, or diameter of dielectric insulator

$\epsilon$  = dielectric constant of insulator ( $\epsilon = \epsilon_r \epsilon_o$ )

$\mu$  = magnetic permeability ( $\mu = \mu_r \mu_o$ )

l = length of the cable



**Figure 49: Coaxial cable cutaway**

## PCB and wire

Table 19: Resistance per length for different wire types (AWG)

AWG	Stds	Outside diameter		Area		dc resistance	
		in	mm	circular mils	mm <sup>2</sup>	Ω / 1000 ft	Ω / km
36	Solid	0.005	0.127	25	0.013	445	1460
36	7/44	0.006	0.152	28	0.014	371	1271
34	Solid	0.0063	0.160	39.7	0.020	280	918
34	7/42	0.0075	0.192	43.8	0.022	237	777
32	Solid	0.008	0.203	67.3	0.032	174	571
32	7/40	0.008	0.203	67.3	0.034	164	538
30	Solid	0.010	0.254	100	0.051	113	365
30	7/38	0.012	0.305	112	0.057	103	339
28	Solid	0.013	0.330	159	0.080	70.8	232
28	7/36	0.015	0.381	175	0.090	64.9	213
26	Solid	0.016	0.409	256	0.128	43.6	143
26	10/36	0.021	0.533	250	0.128	41.5	137
24	Solid	0.020	0.511	404	0.205	27.3	89.4
24	7/32	0.024	0.610	448	0.229	23.3	76.4
22	Solid	0.025	0.643	640	0.324	16.8	55.3
22	7/30	0.030	0.762	700	0.357	14.7	48.4
20	Solid	0.032	0.813	1020	0.519	10.5	34.6
20	7/28	0.038	0.965	1111	0.562	10.3	33.8
18	Solid	0.040	1.020	1620	0.823	6.6	21.8
18	7/26	0.048	1.219	1770	0.902	5.9	19.2
16	Solid	0.051	1.290	2580	1.310	4.2	13.7
16	7/24	0.060	1.524	2828	1.442	3.7	12.0
14	Solid	0.064	1.630	4110	2.080	2.6	8.6
14	7/22	0.073	1.854	4480	2.285	2.3	7.6

Table 20: Maximum current vs. AWG

Wire gauge	Polyethylene Neoprene Polyvinylchloride (semi-ridged) at 80°C	Polypropylene Polyethylene (high density) at 90°C	Polyvinylchloride Nylon at 105°C	Kynar Polyethylene Thermoplastic at 125°C	Kapton Teflon Silicon at 200°C
AWG	$I_{max}$ (A)	$I_{max}$ (A)	$I_{max}$ (A)	$I_{max}$ (A)	$I_{max}$ (A)
30	2	3	3	3	4
28	3	4	4	5	6
26	4	5	5	6	7
24	6	7	7	8	10
22	8	9	10	11	13
20	10	12	13	14	17
18	15	17	18	20	24
16	19	22	24	26	32
14	27	30	33	40	45
12	36	40	45	50	55
10	47	55	58	70	75
Note: Wire is in free air at 25°C					

**Example**

What is the maximum current that can be applied to a 30 gauge Teflon wire in a room temperature environment? What will the self-heating be?

**Answer**

$$I_{max} = 4A$$

Wire temperature = 200°C



- Thermistor
- Resistive temperature detector (RTD)
- Diode temperature characteristics
- Thermocouple (J and K)



Table 21: Temperature sensor overview

	Thermistor	RTD	Diode	Thermocouple
<b>Temp range</b>	-55°C < T < 150°C	-200°C < T < 850°C	-55°C < T < 150°C	-250°C < T < 1800°C
<b>Cost</b>	Low	High	Low	Low
<b>Accuracy</b>	Good accuracy at one temperature. Less accurate over full range.	Excellent accuracy	Poor accuracy without calibration.	Good accuracy with polynomial correction.
<b>Linearity</b>	Very nonlinear. Follows reciprocal of logarithmic function.	Fairly linear Nonlinearity < 4.5% of full scale. Relatively simple quadratic function.	Fairly linear Slope ≈ -2mV/°C Slope varies according to current excitation, diode type, and diode processing.	Fairly linear Nonlinearity < 10% of full scale Complex 10th order polynomial
<b>Construction</b>	Less rugged	Depends on Type (can be rugged)	Rugged	Most rugged
<b>Output range</b>	Typically 10s to 100s of kΩ full scale. Very wide variation in resistance.	18 to 390 Ω for PT100 180 to 3.9 kΩ for-PT1000	0.4 to 0.8V	10s of millivolts
<b>Applications</b>	General purpose	Scientific and industrial	Low cost temperature monitor Low cost linear response	Industrial temperature measurement
<b>General</b>	Requires excitation	Requires excitation	Requires excitation	Self-powered Requires cold junction comp

**Thermistor: Resistance to temperature, Steinhart-Hart equation**

$$\frac{1}{T} = a + b \ln(R) + c (\ln(R))^3 \quad (87) \quad \text{Convert resistance to temperature for a thermistor}$$

Where

T = temperature in Kelvin

a, b, c = Steinhart-Hart equation constants

R = resistance in ohms

**Thermistor: Temperature to resistance, Steinhart-Hart equation**

$$R = \exp\left(x - \frac{y}{2}\right)^{\frac{1}{3}} - \exp\left(x - \frac{y}{2}\right)^{\frac{1}{3}} \quad (88) \quad \text{Convert temperature to resistance for a thermistor}$$

$$y = \frac{a - \frac{1}{T}}{c} \quad (89) \quad \text{Factor used in Equation 88}$$

$$x = \sqrt{\left(\frac{b}{3c}\right)^3 + \frac{y^2}{4}} \quad (90) \quad \text{Factor used in Equation 88}$$

Where

R = resistance in ohms

T = temperature in Kelvin

a, b, c = Steinhart-Hart equation constants

x, y = Steinhart-Hart factors used in temperature to resistance equation

## RTD equation temperature to resistance

$$R_{\text{rtd}} = R_0[1 + A_0T + B_0T^2 + C_0(T - 100)T^3] \quad (91) \quad \text{RTD resistance for } T < 0^\circ\text{C}$$

$$R_{\text{rtd}} = R_0[1 + A_0T + B_0T^2] \quad (92) \quad \text{RTD resistance for } T > 0^\circ\text{C}$$

Where

$R_{\text{rtd}}$  = resistance of RTD over temperature range of  $(-200^\circ\text{C} < T < 850^\circ\text{C})$

$R_0 = 100 \Omega$  for PT-100,  $1000 \Omega$  for PT-1000

$A_0, B_0, C_0$  = Callendar-Van Dusen coefficients

$T$  = temperature in degrees Celsius ( $^\circ\text{C}$ )

## RTD equation resistance to temperature ( $T > 0^\circ\text{C}$ )

$$T = \frac{-A_0 + \sqrt{A_0^2 + 4B_0\left(1 - \frac{R_{\text{RTD}}}{R_0}\right)}}{2B_0} \quad (93) \quad \text{RTD resistance for } T > 0^\circ\text{C}$$

Where

$R_{\text{RTD}}$  = resistance of RTD over temperature range of  $(-200^\circ\text{C} < T < 850^\circ\text{C})$

$R_0 = 100 \Omega$

$A_0, B_0, C_0$  = Callendar-Van Dusen coefficients

$T$  = temperature in degrees Celsius ( $^\circ\text{C}$ )

**Table 22: Callendar-Van Dusen coefficients for different RTD standards**

	IEC-751 DIN 43760 BS 1904 ASTM-E1137 EN-60751	JISC 1604	US Industrial Standard D-100 American	US Industrial Standard American	ITS-90
<b>A0</b>	+3.9083E-3	+3.9739E-3	+3.9787E-3	+3.9692E-3	+3.9888E-3
<b>B0</b>	-5.775E-7	-5.870E-7	-5.8686E-7	-5.8495E-7	-5.915E-7
<b>C0</b>	-4.183E-12	-4.4E-12	-4.167E-12	-4.233E-12	-3.85E-12

## Example

What is the temperature given an ITS-90 PT100 resistance of  $120 \Omega$ ?

## Answer

$$T = \frac{-(3.9888 \cdot 10^{-3}) + \sqrt{(3.9888 \cdot 10^{-3})^2 + 4(-5.915 \cdot 10^{-7})\left(1 - \frac{120}{100}\right)}}{2(-5.915 \cdot 10^{-7})} = 50.5^\circ\text{C}$$



**RTD equation resistance to temperature (T<0°C)**

$$T = \sum_{i=0}^n \alpha_i (R_{rtd})^i \tag{94}$$

RTD resistance for T<0°C

Where

T = temperature in degrees Celsius (°C)

R<sub>RTD</sub> = resistance of RTD over temperature range of (T<0°C)

α<sub>i</sub> = polynomial coefficients for converting RTD resistance to temperature for T<0°C

**Table 23: Coefficients for 5<sup>th</sup> order RTD resistance to temperature**

	IEC-751 DIN 43760 BS 1904 ASTM-E1137 EN-60751	JISC 1604	US Industrial Standard D-100 American	US Industrial Standard American	ITS-90
α0	-2.4202E+02	-2.3820E+02	-2.3818E+02	-2.3864E+02	-2.3791E+02
α1	2.2228E+00	2.1898E+00	2.1956E+00	2.1973E+00	2.2011E+00
α2	2.5857E-03	2.5226E-03	2.4413E-03	2.4802E-03	2.3223E-03
α3	-4.8266E-06	-4.7825E-06	-4.7517E-06	-4.7791E-06	-4.6280E-06
α4	-2.8152E-08	-2.7009E-08	-2.3831E-08	-2.5157E-08	-1.9702E-08
α5	1.5224E-10	1.4719E-10	1.3492E-10	1.4020E-10	1.1831E-10

**Example**

Find the temperature given an ITS-90 PT100 resistance of 60 Ω.

**Answer**

$$T = (-2.3791E + 02) * (60)^0 + (2.2011E + 00) * (60)^1 + (2.3223E - 03) * (60)^2 + \dots + (2.3223E - 03) * (60)^5 = -98.6^\circ\text{C}$$

### Diode equation vs. temperature

$$V_D = \frac{nkT}{q} \ln\left(\frac{I}{I_S} + 1\right) \approx \frac{nkT}{q} \ln\left(\frac{I}{I_S}\right) \quad (95) \text{ Diode voltage}$$

Where

$V_D$  = diode voltage vs. temperature and current

$n$  = diode ideality factor (ranges from 1 to 2)

$k = 1.38 \times 10^{-23}$  J/K, Boltzmann's constant

$T$  = temperature in Kelvin

$q = 1.60 \times 10^{-19}$  C, charge of an electron

$I$  = forward diode current in amps

$I_S$  = saturation current

$$I_S = \alpha T^{(3/n)} \exp\left(-\frac{qV_G}{nkT}\right) \quad (96) \text{ Saturation current}$$

Where

$I_S$  = saturation current

$\alpha$  = constant related to the cross sectional area of the junction

$V_G$  = diode voltage vs. temperature and current

$n$  = diode ideality factor (ranges from 1 to 2)

$k = 1.38 \times 10^{-23}$  J/K, Boltzmann's constant

$T$  = temperature in Kelvin

$q = 1.60 \times 10^{-19}$  C, charge of an electron

## Diode voltage versus temperature

Figure 50 shows an example of the temperature drift for a diode. Depending on the characteristics of the diode and the forward current the slope and offset of this curve will change. However, typical diode drift is about  $-2\text{mV}/^\circ\text{C}$ . A forward drop of about  $0.6\text{V}$  is typical for room temperature.

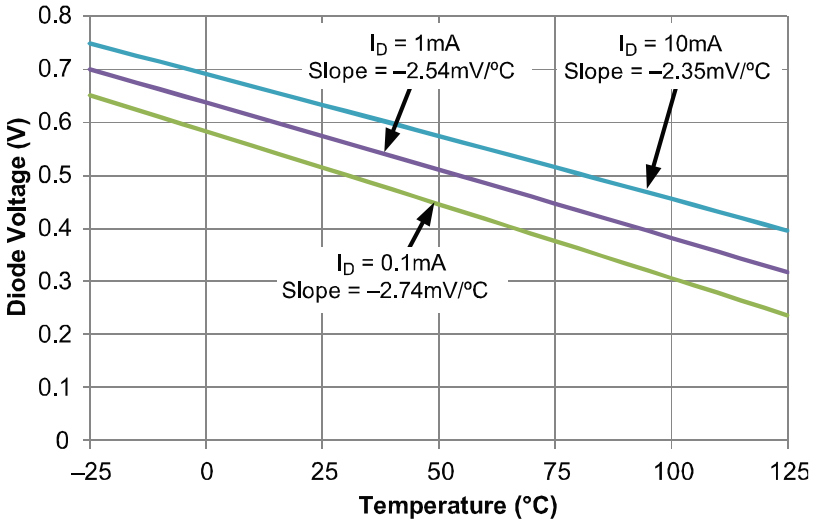


Figure 50: Diode voltage drop vs. temperature

## Type J thermocouples translating temperature to voltage (ITS-90 standard)

$$V_t = \sum_{i=0}^n c_i (T)^i \quad (97) \quad \text{Thermoelectric voltage}$$

Where

$V_T$  = thermoelectric voltage

$T$  = temperature in degrees Celsius

$c_i$  = translation coefficients

**Table 24: Type J thermocouple temperature to voltage coefficients**

Type J thermocouple temperature to voltage		
	-219°C to 760°C	760°C to 1,200°C
<b>c0</b>	0.0000000000E+00	2.9645625681E+05
<b>c1</b>	5.0381187815E+01	-1.4976127786E+03
<b>c2</b>	3.0475836930E-02	3.1787103924E+00
<b>c3</b>	-8.5681065720E-05	-3.1847686701E-03
<b>c4</b>	1.3228195295E-07	1.5720819004E-06
<b>c5</b>	-1.7052958337E-10	-3.0691369056E-10
<b>c6</b>	2.0948090697E-13	--
<b>c7</b>	-1.2538395336E-16	--
<b>c8</b>	1.5631725697E-20	--

## Type J thermocouples translating voltage to temperature (ITS-90 standard)

$$T = \sum_{i=0}^n c_i (V_T)^i \quad (98) \quad \text{Temperature}$$

Table 25: Type J thermocouple voltage to temperature coefficients

Type J thermocouple voltage to temperature			
	-219°C to 0°C	0°C to 760°C	760°C to 1,200°C
<b>c0</b>	0.000000000E+00	0.000000000E+00	-3.113581870E+03
<b>c1</b>	1.952826800E-02	1.978425000E-02	3.005436840E-01
<b>c2</b>	-1.228618500E-06	-2.001204000E-07	-9.947732300E-06
<b>c3</b>	-1.075217800E-09	1.036969000E-11	1.702766300E-10
<b>c4</b>	-5.908693300E-13	-2.549687000E-16	-1.430334680E-15
<b>c5</b>	-1.725671300E-16	3.585153000E-21	4.738860840E-21
<b>c6</b>	-2.813151300E-20	-5.344285000E-26	--
<b>c7</b>	-2.396337000E-24	5.099890000E-31	--
<b>c8</b>	-8.382332100E-29	--	--

## Type K thermocouples translating temperature to voltage (ITS-90 standard)

$$V_T = \sum_{i=0}^n c_i (T)^i \quad (99) \quad \text{Thermoelectric voltage for } T < 0^\circ\text{C}$$

$$V_t = \left[ \sum_{i=0}^n c_i (T)^i \right] + \alpha_0 e^{[\alpha_1 (T - 126.9686)]^2} \quad (100) \quad \text{Thermoelectric voltage for } T > 0^\circ\text{C}$$

Where

$V_T$  = thermoelectric voltage

$T$  = temperature in degrees Celsius

$c_i$  = translation coefficients

$\alpha_0, \alpha_1$  = translation coefficients

**Table 26: Type K thermocouple temperature to voltage coefficients**

	-219°C to 760°C	760°C to 1,200°C
$c_0$	0.0000000000E+00	-1.7600413686E+01
$c_1$	3.9450128025E+01	3.8921204975E+01
$c_2$	2.3622373598E-02	1.8558770032E-02
$c_3$	-3.2858906784E-04	-9.9457592874E-05
$c_4$	-4.9904828777E-06	3.1840945719E-07
$c_5$	-6.7509059173E-08	-5.6072844889E-10
$c_6$	-5.7410327428E-10	5.6075059059E-13
$c_7$	-3.1088872894E-12	-3.2020720003E-16
$c_8$	-1.0451609365E-14	9.7151147152E-20
$c_9$	-1.9889266878E-17	-1.2104721275E-23
$c_{10}$	-1.6322697486E-20	--
$\alpha_0$	--	1.1859760000E+02
$\alpha_1$	--	-1.1834320000E-04

## Type K thermocouples translating voltage to temperature (ITS-90 standard)

$$T = \sum_{i=0}^n c_i (V_t)^i \quad (101) \text{ Temperature}$$

Table 27: Type K thermocouple voltage to temperature coefficients

	-219°C to 0°C	0°C to 760°C	760°C to 1,200°C
<b>c0</b>	0.0000000E+00	0.0000000E+00	-1.3180580E+02
<b>c1</b>	2.5173462E-02	2.5083550E-02	4.8302220E-02
<b>c2</b>	-1.1662878E-06	7.8601060E-08	-1.6460310E-06
<b>c3</b>	-1.0833638E-09	-2.5031310E-10	5.4647310E-11
<b>c4</b>	-8.9773540E-13	8.3152700E-14	-9.6507150E-16
<b>c5</b>	-3.7342377E-16	-1.2280340E-17	8.8021930E-21
<b>c6</b>	-8.6632643E-20	9.8040360E-22	-3.1108100E-26
<b>c7</b>	-1.0450598E-23	-4.4130300E-26	--
<b>c8</b>	-5.1920577E-28	1.0577340E-30	--
<b>c9</b>	--	-1.0527550E-35	--

**Table 28: Seebeck coefficients for different material**

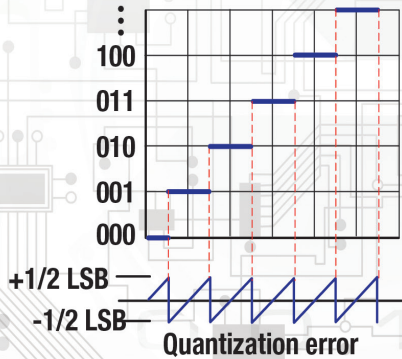
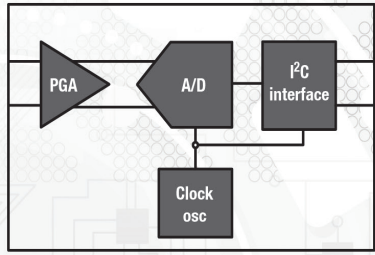
<b>Material</b>	<b>Seebeck coefficient</b>	<b>Material</b>	<b>Seebeck coefficient</b>	<b>Material</b>	<b>Seebeck coefficient</b>
Aluminum	3.5	Gold	6.5	Rhodium	6.0
Antimony	47	Iron	19	Selenium	900
Bismuth	-72	Lead	4.0	Silicon	440
Cadmium	7.5	Mercury	0.6	Silver	6.5
Carbon	3.0	Nichrome	25	Sodium	-2.0
Constantan	-35	Nickel	-15	Tantalum	4.5
Copper	6.5	Platinum	0	Tellurium	500
Germanium	300	Potassium	-9.0	Tungsten	7.5

Note: Units are  $\mu\text{V}/^\circ\text{C}$ . All data at temperature of  $0^\circ\text{C}$



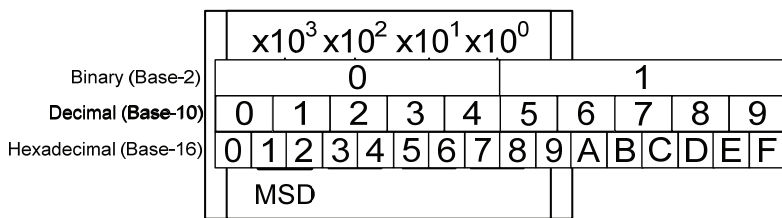
# A/D conversion

- Binary/hex conversions
- A/D and D/A transfer function
- Quantization error
- Signal-to-noise ratio (SNR)
- Signal-to-noise and distortion (SINAD)
- Total harmonic distortion (THD)
- Effective number of bits (ENOB)
- Noise-free resolution and effective resolution



# A/D conversion

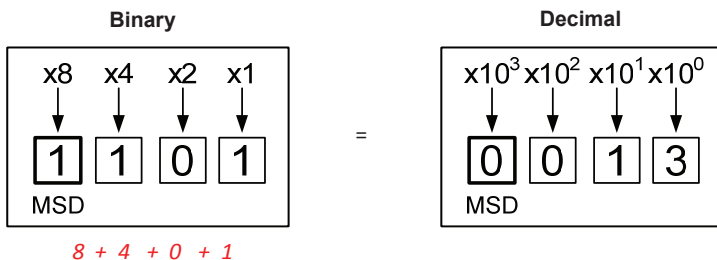
## Numbering systems: Binary, decimal, and hexadecimal



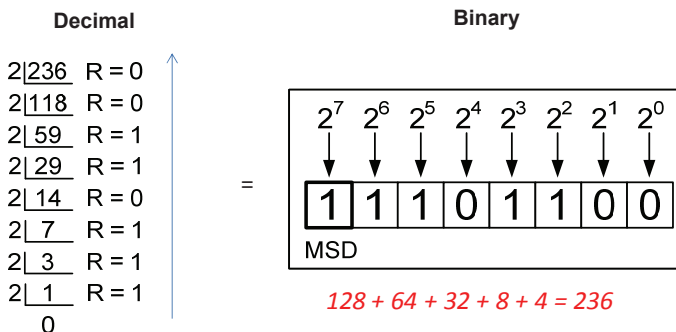
$$2(1000) + 3(100) + 4(10) + 1(1) = 2,341$$

MSD = Most significant digit

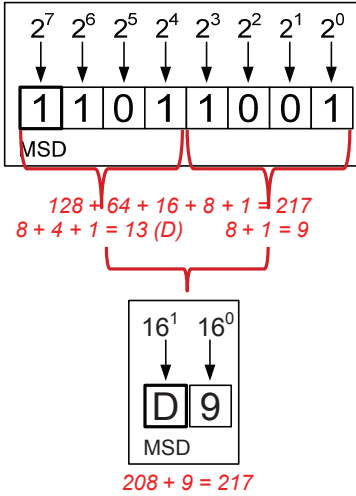
### Example conversion: Binary to decimal



### Example conversion: Decimal to binary

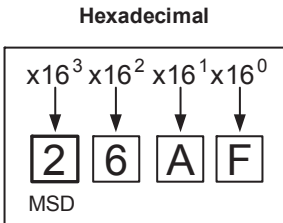


**Example conversion: Binary to hexadecimal**

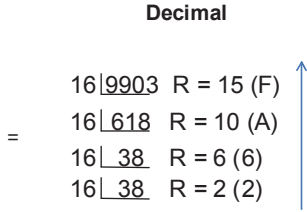


**Example Conversion: Hexadecimal to binary**

Decimal (Base-10)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal (Base-16)	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F



$2(4096) + 6(256) + 10(16) + 16(1) = 9903$

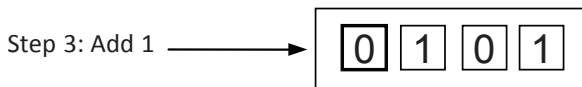
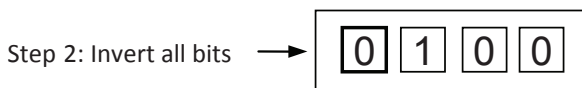
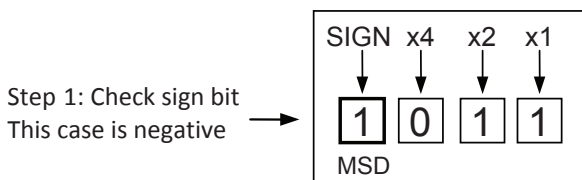


## A/D conversion

Table 29: Different data formats

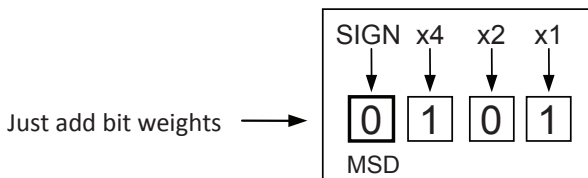
Code	Straight binary	Offset binary	2's complement
Binary	Decimal value	Decimal value	Decimal value
11111111	255	127	-1
11000000	192	64	-64
10000000	128	0	-128
01111111	127	-1	127
01000000	64	-64	64
00000000	0	-128	0

### Converting two's complement to decimal: Negative number example



Final result →  $-(4+1) = -5$

### Converting two's complement to decimal: Positive number example



Final result →  $4+1 = 5$

Table 30: LSB voltage vs. resolution and reference voltage

		Reference voltage			
		1.024	1.25	2.048	2.5
Resolution	8	4 mV	4.88 mV	8 mV	9.76 mV
	10	1 mV	1.22 mV	2 mV	2.44 mV
	12	250 $\mu$ V	305 $\mu$ V	500 $\mu$ V	610 $\mu$ V
	14	52.5 $\mu$ V	76.3 $\mu$ V	125 $\mu$ V	152.5 $\mu$ V
	16	15.6 $\mu$ V	19.1 $\mu$ V	31.2 $\mu$ V	38.14 $\mu$ V
	18	3.91 $\mu$ V	4.77 $\mu$ V	7.81 $\mu$ V	9.53 $\mu$ V
	20	0.98 $\mu$ V	1.19 $\mu$ V	1.95 $\mu$ V	2.384 $\mu$ V
	22	244 nV	299 nV	488 nV	596 nV
	24	61 nV	74.5 nV	122 nV	149 nV

Table 31: LSB voltage vs. resolution and reference voltage

		Reference voltage			
		3	3.3	4.096	5
Resolution	8	11.7 mV	12.9 mV	16 mV	19.5 mV
	10	2.93 mV	3.222 mV	4 mV	4.882 mV
	12	732 $\mu$ V	806 $\mu$ V	1 mV	1.221 mV
	14	183 $\mu$ V	201 $\mu$ V	250 $\mu$ V	305 $\mu$ V
	16	45.77 $\mu$ V	50.35 $\mu$ V	62.5 $\mu$ V	76.29 $\mu$ V
	18	11.44 $\mu$ V	12.58 $\mu$ V	15.6 $\mu$ V	19.07 $\mu$ V
	20	2.861 $\mu$ V	3.147 $\mu$ V	3.91 $\mu$ V	4.768 $\mu$ V
	22	715 nV	787 nV	976 nV	1.192 $\mu$ V
	24	179 nV	196 nV	244 nV	298 nV

# A/D conversion

## DAC definitions

Resolution = $n$	The number of bits used to quantify the output
Codes = $2^n$	The number of input code combinations
Reference voltage = $V_{REF}$	Sets the LSB voltage or current size and converter range
LSB = $V_{REF} / 2^n$	The output voltage or current step size of each code
Full-scale code = $2^n - 1$	The largest code that can be written
Full-scale voltage = $V_{REF} - 1\text{LSB}$	Full-scale output voltage of the DAC
Transfer function = $V_{REF} \times (\text{code} / 2^n)$	Relationship between input code and output voltage or current

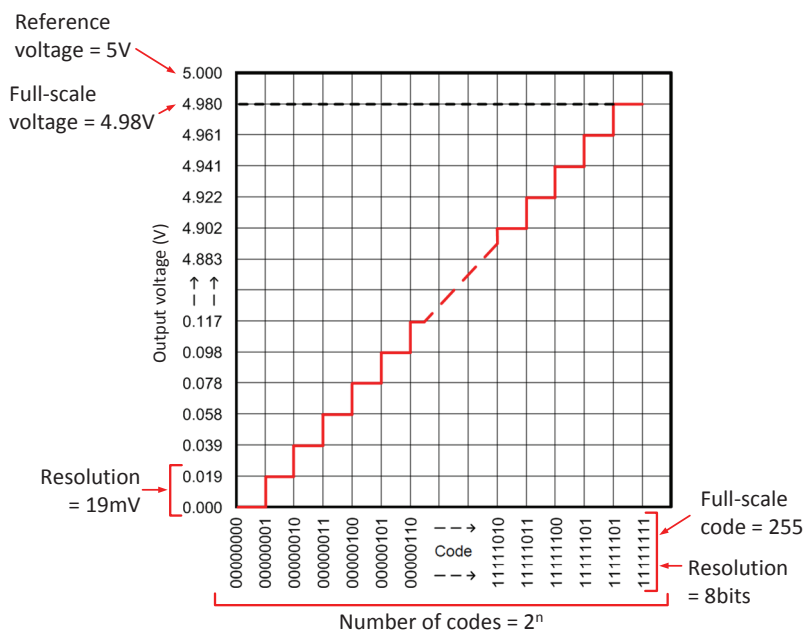


Figure 51: DAC transfer function

ADC definitions

Resolution = n	The number of bits used to quantify the output
Codes = $2^n$	The number of input code combinations
Reference voltage = $V_{REF}$	Sets the LSB voltage or current size and converter range
LSB = $V_{REF} / (2^n - 1)$	The voltage step size of each code. Note that some topologies may use $2^n$ as opposed to $2^n - 1$ in the denominator.
Full-scale code = $2^n - 1$	The largest code that can be written.
Full-scale voltage = $V_{REF}$	Full-scale output voltage of the DAC. Note that the full-scale voltage will differ if the alternative definition for resolution is used.
Transfer function = $V_{REF} \times (\text{code} / 2^n)$	Relationship between input code and output voltage or current

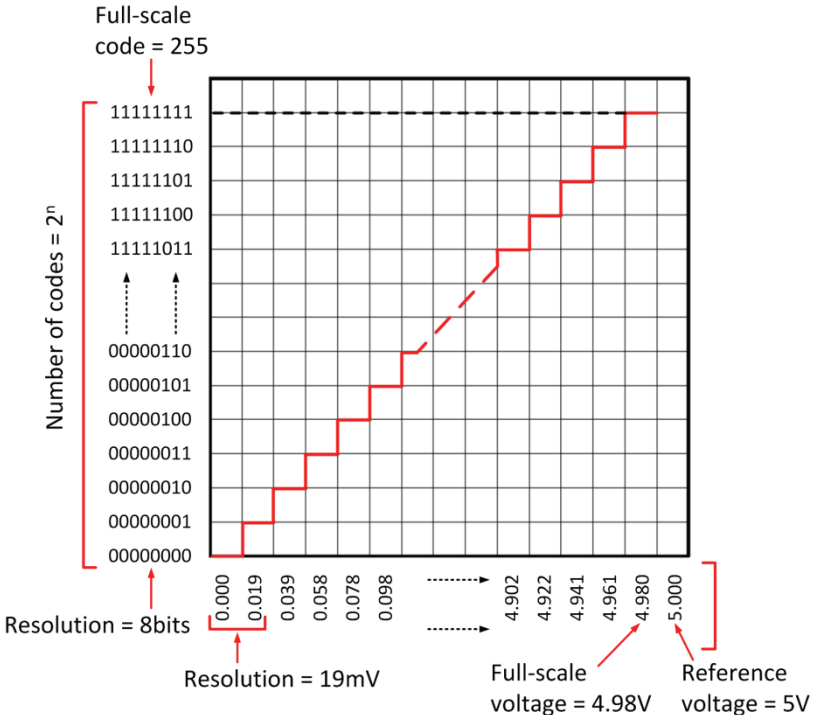


Figure 52: ADC transfer function

## A/D conversion

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### Quantization error of ADC

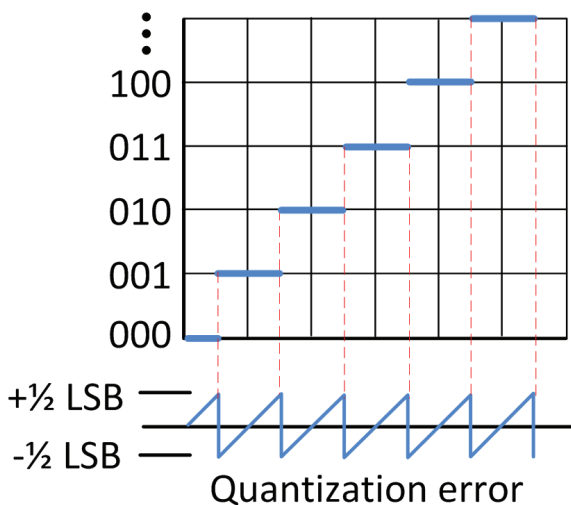


Figure 53: Quantization error of an A/D converter

### Quantization error

The error introduced as a result of the quantization process. The amount of this error is a function of the resolution of the converter. The quantization error of an A/D converter is  $1/2 \text{ LSB}$ . The quantization error signal is the difference between the actual voltage applied and the ADC output (Figure 53). The rms of the quantization signal is  $1\text{LSB}/\sqrt{12}$



**Signal-to-noise ratio (SNR) from quantization noise only**

$$\text{MaxRMSSignal} = \frac{\text{FSR}/2}{\sqrt{2}} = \frac{1\text{LSB} \times 2^{N-1}}{\sqrt{2}} \quad (102)$$

$$\text{RMSNoise} = \frac{1\text{LSB}}{\sqrt{12}} \text{ from quantization noise} \quad (103)$$

$$\text{SNR} = \frac{\text{MaxRMSSignal}}{\text{RMSNoise}} = \frac{1\text{LSB} \times 2^{N-1}/\sqrt{2}}{1\text{LSB}/\sqrt{12}} = 2^{N-1}\sqrt{6} \quad (104)$$

$$\text{SNR(dB)} = 20\log(\text{SNR}) = [20 \log(2)]N + 20\log\left(\frac{\sqrt{6}}{2}\right) \quad (105)$$

$$\text{SNR(dB)} \approx 6.02N + 1.76 \quad (106)$$

Where

FSR = full-scale range of the A/D converter

1LSB = the voltage of 1LSB,  $V_{\text{REF}}/2^n$

N = the resolution of the A/D converter

MaxRMSSignal = the rms equivalent of the ADC's full-scale input

RMSNoise = the rms noise from quantization

SNR = the ratio of rms signal to rms noise

**Example**

What is the SNR for an 8-bit A/D converter with 5V reference, assuming only quantization noise?

**Answer**

$$\text{SNR} = 2^{N-1}\sqrt{6} = 2^{8-1}\sqrt{6} = 314$$

$$\text{SNR(dB)} = 20\log(314) = 49.9 \text{ dB}$$

$$\text{SNR(dB)} = 6.02(8) + 1.76 = 49.9 \text{ dB}$$

## A/D conversion

---

### Total harmonic distortion (Vrms)

$$\text{THD} = \frac{(\text{RMSDistortion})}{(\text{MaxRMSSignal})} = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \dots + V_n^2}}{V_1} \quad (107)$$

$$\text{THD(dB)} = 20\log\left(\frac{\text{RMSDistortion}}{\text{MaxRMSSignal}}\right) \quad (108)$$

Where

THD = total harmonic distortion, the ratio of the rms distortion to the rms signal

RMSDistortion = the rms sum of all harmonic components

MaxRMSSignal = the rms value of the input signal

$V_1$  = the fundamental, generally the input signal

$V_2, V_3, V_4, \dots, V_n$  = harmonics of the fundamental

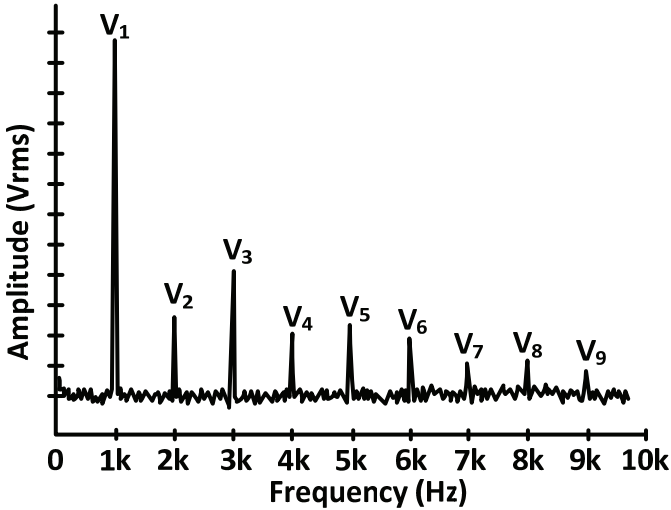


Figure 54: Fundamental and harmonics in Vrms

## Total harmonic distortion (dBc)

$$\text{THD} = 10 \log \left[ 10^{\left(\frac{D_2}{10}\right)} + 10^{\left(\frac{D_3}{10}\right)} + 10^{\left(\frac{D_4}{10}\right)} + \dots + 10^{\left(\frac{D_n}{10}\right)} \right] \quad (109)$$

Where

THD = total harmonic distortion. The ratio of the rms distortion to the rms signal  
 $D_1$  = the fundamental, generally the input signal. This is normalized to 0 dBc  
 $D_2, D_3, D_4, \dots, D_n$  = harmonics of the fundamental measured relative to the fundamental

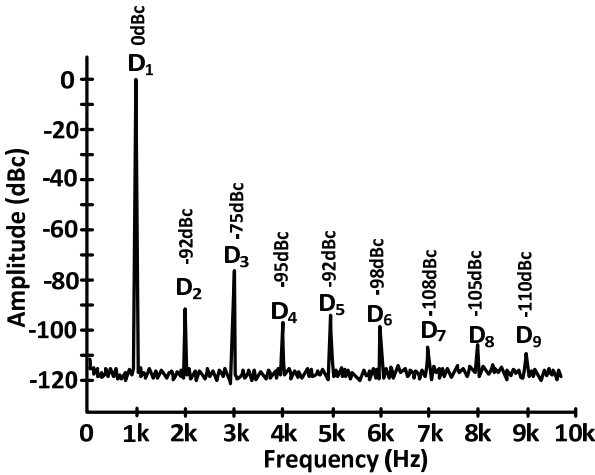


Figure 55: Fundamental and harmonics in dBc

## Example

Determine THD for the example above.

## Answer

$$\text{THD} = 10 \log \left[ 10^{\left(\frac{-92}{10}\right)} + 10^{\left(\frac{-75}{10}\right)} + 10^{\left(\frac{-95}{10}\right)} + \dots + 10^{\left(\frac{-110}{10}\right)} \right]$$

$$\text{THD} = -74.76 \text{ dB}$$

## A/D conversion

---

### Ac signals

#### Signal-to-noise and distortion (SINAD) and effective number of bits (ENOB)

$$\text{SINAD(dB)} = 20 \log \left( \frac{\text{MaxRMSSignal}}{\sqrt{\text{RMSNoise}^2 + \text{RMSDistortion}^2}} \right) \quad (110)$$

$$\text{SINAD(dB)} = -20 \log \left( \sqrt{10^{\left(\frac{-\text{SNR(dB)}}{10}\right)} + 10^{\left(\frac{\text{THD(dB)}}{10}\right)}} \right) \quad (111)$$

$$\text{ENOB} = \frac{\text{SINAD(dB)} - 1.76\text{dB}}{6.02} \quad (112)$$

Where

MaxRMSSignal = the rms equivalent of the ADC's full-scale input

RMSNoise = the rms noise integrated across the A/D converters

RMSDistortion = the rms sum of all harmonic components

SINAD = the ratio of the full-scale signal-to-noise ratio and distortion

THD = total harmonic distortion. The ratio of the rms distortion to the rms signal.

SNR = the ratio of rms signal to rms noise

### Example

Calculate the SNR, THD, SINAD and ENOB given the following information:

MaxRMSSignal = 1.76 Vrms

RMSDistortion = 50  $\mu$ Vrms

RMSNoise = 100  $\mu$ Vrms

### Answer

$$\text{SNR(dB)} = 20 \log \left( \frac{1.76 \text{ Vrms}}{100 \mu\text{Vrms}} \right) = 84.9 \text{ dB}$$

$$\text{THD(dB)} = 20 \log \left( \frac{50 \mu\text{Vrms}}{1.76 \text{ Vrms}} \right) = -90.9 \text{ dB}$$

$$\text{SINAD(dB)} = 20 \log \left( \frac{1.76 \text{ V rms}}{\sqrt{(100 \mu\text{Vrms})^2 + (50 \mu\text{Vrms})^2}} \right) = 83.9 \text{ dB}$$

$$\text{SINAD(dB)} = -20 \log \left( \sqrt{10^{\left(\frac{-83.9 \text{ dB}}{10}\right)} + 10^{\left(\frac{-90.9 \text{ dB}}{10}\right)}} \right) = 83.9 \text{ dB}$$

$$\text{ENOB} = \frac{83.9\text{dB} - 1.76\text{dB}}{6.02} = 13.65$$

**Dc signals****Noise free resolution and effective resolution**

$$\text{NoiseFreeResolution} = \log_2 \left( \frac{2^N}{\text{PeaktoPeakNoiseinLSB}} \right) \quad (113)$$

$$\text{EffectiveResolution} = \log_2 \left( \frac{2^N}{\text{rmsNoiseinLSB}} \right) \quad (114)$$

$$\text{PeaktoPeakNoiseinLSB} \approx 6.6 \times \text{rmsNoiseinLSB} \quad (115)$$

$$\text{EffectiveResolution} \approx \text{NoiseFreeResolution} + 2.7 \quad (116)$$

Note: The maximum *effective resolution* is never greater than the ADC resolution. For example, a 24-bit converter cannot have an effective resolution greater than 24 bits.

**Example**

What is the noise-free resolution and effective resolution for a 24-bit converter assuming the peak-to-peak noise is 7 LSBs?

**Answer**

$$\text{NoiseFreeResolution} = \log_2 \left( \frac{2^{24}}{7} \right) = 21.2$$

$$\text{EffectiveResolution} = \log_2 \left( \frac{2^{24}}{6.6} \right) = 23.9$$

$$\text{EffectiveResolution} = 21.2 + 2.7 = 23.9$$

$$\text{Time Constant} = \tau = R C$$

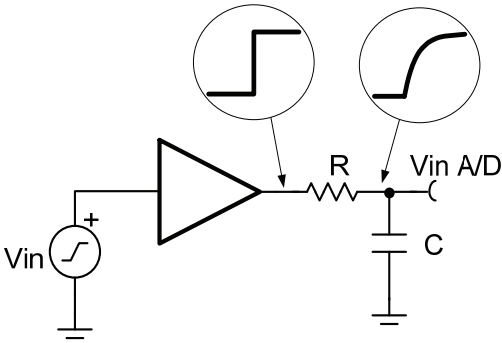


Figure 56: Settling time for RC circuit-related to A/D converters

Table 32: Conversion accuracy achieved after a specified time

Settling time in time constants ( $N_{TC}$ )	Accuracy in bits	Settling time in time constants ( $N_{TC}$ )	Accuracy in bits
1	1.44	10	14.43
2	2.89	11	15.87
3	4.33	12	17.31
4	5.77	13	18.76
5	7.21	14	20.20
6	8.66	15	21.64
7	10.10	16	23.08
8	11.54	17	24.53
9	12.98	18	25.97

$$N = \log_2(e^{-N_{TC}}) \quad (117)$$

Where

$N$  = the number of bits of accuracy the RC circuit has settled to after  $N_{TC}$  number of time constants.

$N_{TC}$  = the number of RC time constants

**Table 33: Time required to settle to a specified conversion accuracy**

Accuracy in bits (N)	Settling time in time constants ( $N_{TC}$ )	Accuracy in bits (N)	Settling time in time constants ( $N_{TC}$ )
8	5.55	17	11.78
9	6.24	18	12.48
10	6.93	19	13.17
11	7.62	20	13.86
12	8.32	21	14.56
13	9.01	22	15.25
14	9.70	23	15.94
15	10.40	24	16.64
16	11.09	25	17.33

$$N_{TC} = \ln(2^N) \quad (118)$$

Where

$N_{TC}$  = the number of time constants required to achieve N bits of settling

N = the number of bits of accuracy





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$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$      $\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$      $\operatorname{tg} h x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$w = c + di$

$v = (a-c, b-d)$      $\operatorname{sh} w = \frac{e^w - e^{-w}}{2}$

$z = a + bi$      $\bar{z} = a - bi$

$\rho = \sqrt{a^2 + b^2}$      $\operatorname{sh} z = \frac{e^z - e^{-z}}{2}$      $z = a + bi$      $\bar{z} = a - bi$

$z = \rho e^{i\theta}$      $\bar{z} = \rho e^{-i\theta}$

$z + \bar{z} = 2 \operatorname{Re}(z)$

$z - \bar{z} = 2i \operatorname{Im}(z)$

$z \bar{z} = |z|^2$      $|z| = \operatorname{Mod} de z$

Si  $z$  es raíz de un Pol.

$z = 2k\pi i, k \in \mathbb{Z}$

$|z| > 0$      $|z| = 0$  ou  $z = 0$

$|z| + |w| \geq |z + w|$

$\operatorname{Re}(z) < |z|$

$|z \cdot w| = |z| \cdot |w|$

$|z/\bar{z}| = |z|/|z| = 1$

$\operatorname{sh} z = \frac{e^z - e^{-z}}{2}$

$\operatorname{tg} h z = \frac{e^z - e^{-z}}{e^z + e^{-z}}$

$\operatorname{cos} \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$\operatorname{sen} \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

$e^a \cdot z \in \mathbb{C}$      $e^b = e$

